

Intermediate Microeconomics

Step by Step

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Preamble

These notes accompany the [Intermediate Microeconomics](#) course at the [University of Alicante](#).

Their purpose is to give you, worked out from start to finish, the calculations that come up again and again in the course. Each chapter takes concrete examples and develops them step by step, so that you can follow the mechanics with pencil and paper and check your own results.

The notes focus on the *how*, not the *why*: the economic intuition and the interpretation of the results are covered in class and in the textbook. What you will find here are the computations.

AI disclosure

These notes were prepared with the assistance of Claude Code (version 2.1.296), Anthropic's coding agent, running the Claude Opus 5.5 model (`claude-opus-5-5`), in October 2026. Working under my direction, the AI drafted the text, carried out the derivations, wrote the code that draws the figures, and built this website. I chose the examples and the parameter values, specified the content and structure of each chapter, and reviewed the material.

— Eduardo Zambrano

Chapter 1

Step by Step: The Consumer's Optimal Choice

1.1 Setup

A consumer chooses a bundle (x, y) of two goods. Preferences are represented by a utility function:

$$u(x, y)$$

The price of good x is p_x and the price of good y is p_y :

$$p_x = 4 \qquad p_y = 1$$

The consumer's income is I :

$$I = 36$$

The budget constraint is:

$$p_x x + p_y y = I \implies 4x + y = 36$$

The consumer chooses the bundle that maximizes utility subject to the budget constraint.

Three preferences:

$$u = x^{1/3}y^{2/3} \qquad u = \frac{1}{3}x + \frac{2}{3}y \qquad u = \min \left\{ \frac{1}{3}x, \frac{2}{3}y \right\}$$

1.2 Cobb-Douglas: $u = x^{1/3}y^{2/3}$

1.2.1 Tangency condition

Marginal utilities:

$$MU_x = \frac{1}{3}x^{-2/3}y^{2/3} \qquad MU_y = \frac{2}{3}x^{1/3}y^{-1/3}$$

Dividing:

$$MRS = \frac{MU_x}{MU_y} = \frac{y}{2x}$$

Setting equal to p_x/p_y :

$$\frac{y}{2x} = \frac{4}{1} \implies y = 8x$$

1.2.2 Optimal choice

Substituting $y = 8x$ into the budget constraint:

$$4x + 8x = 36$$

Solving:

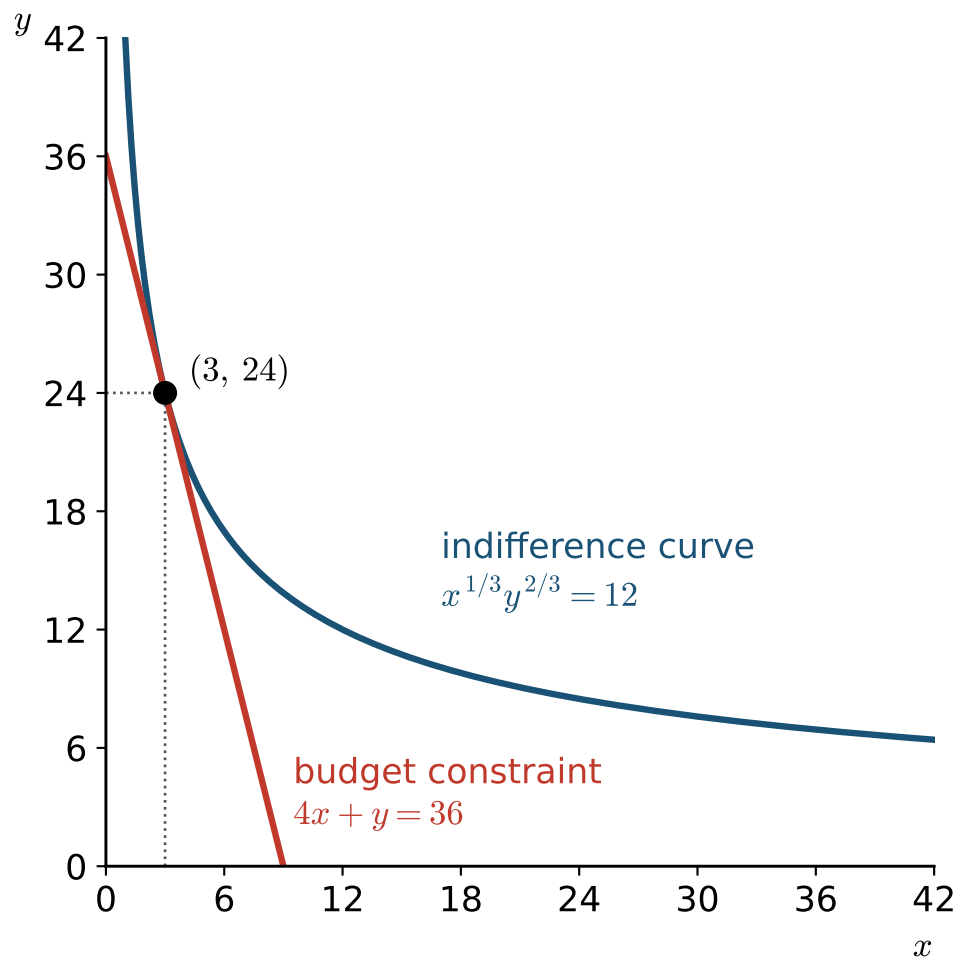
$$x = 3 \qquad y = 8 \cdot 3 = 24$$

1.2.3 Utility attained

Evaluating at $(3, 24)$:

$$u(3, 24) = 3^{1/3} \cdot 24^{2/3} = 3^{1/3} \cdot 4 \cdot 3^{2/3} = 12$$

1.2.4 Indifference curve and budget constraint



1.3 Perfect substitutes: $u = \frac{1}{3}x + \frac{2}{3}y$

1.3.1 Comparing MRS and p_x/p_y

Marginal utilities:

$$MU_x = \frac{1}{3} \quad MU_y = \frac{2}{3}$$

Dividing:

$$MRS = \frac{1/3}{2/3} = \frac{1}{2} \quad \frac{p_x}{p_y} = 4$$

Comparing:

$$MRS < \frac{p_x}{p_y} \implies x = 0$$

1.3.2 Optimal choice

Substituting $x = 0$ into the budget constraint:

$$4 \cdot 0 + y = 36$$

Solving:

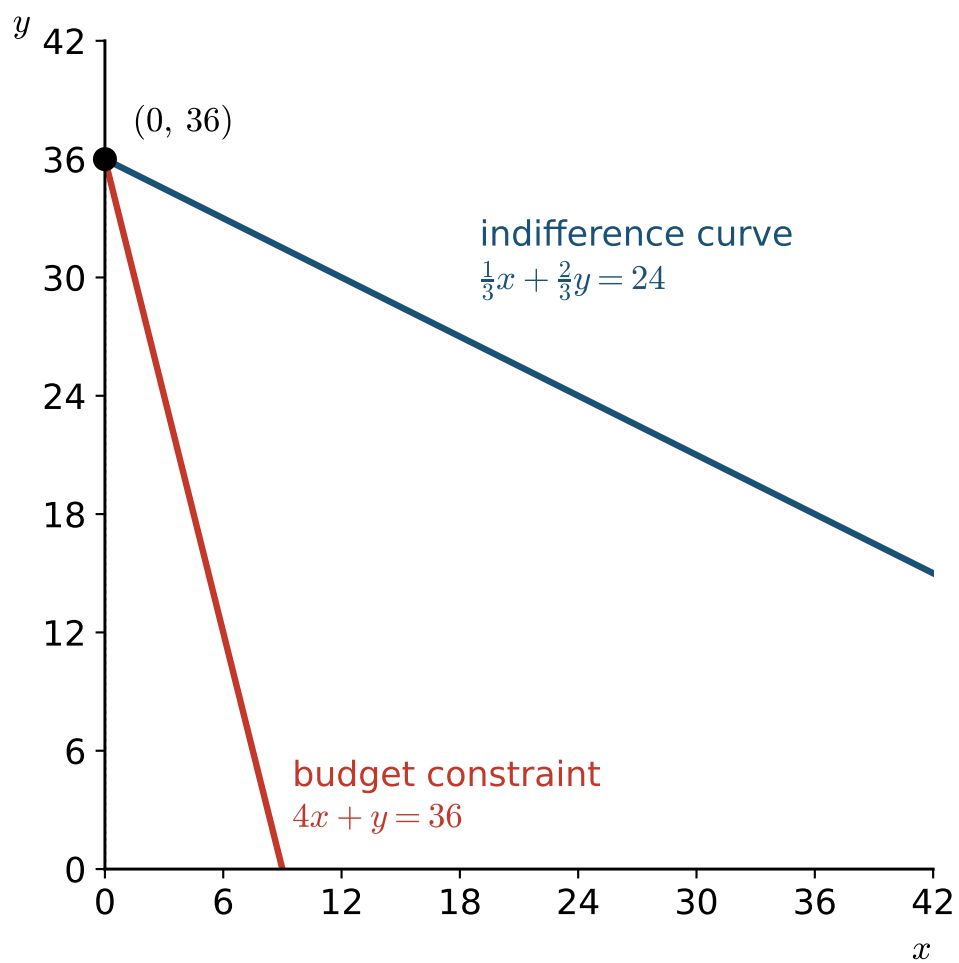
$$x = 0 \qquad y = 36$$

1.3.3 Utility attained

Evaluating at $(0, 36)$:

$$u(0, 36) = \frac{1}{3} \cdot 0 + \frac{2}{3} \cdot 36 = 24$$

1.3.4 Indifference curve and budget constraint



1.4 Perfect complements: $u = \min \left\{ \frac{1}{3}x, \frac{2}{3}y \right\}$

1.4.1 Fixed-proportions condition

Without wasting either good:

$$\frac{1}{3}x = \frac{2}{3}y \implies x = 2y$$

1.4.2 Optimal choice

Substituting $x = 2y$ into the budget constraint:

$$4 \cdot 2y + y = 36 \implies 9y = 36$$

Solving:

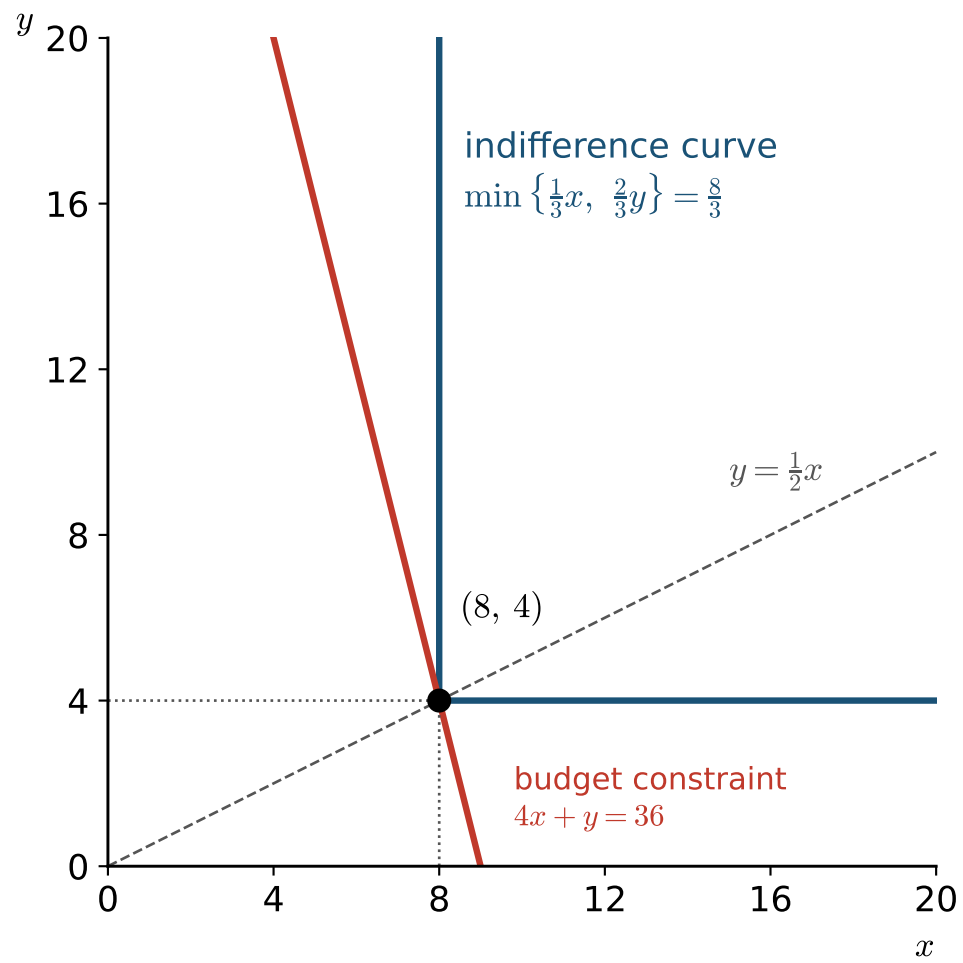
$$y = 4 \qquad x = 2 \cdot 4 = 8$$

1.4.3 Utility attained

Evaluating at $(8, 4)$:

$$u(8, 4) = \min \left\{ \frac{1}{3} \cdot 8, \frac{2}{3} \cdot 4 \right\} = \min \left\{ \frac{8}{3}, \frac{8}{3} \right\} = \frac{8}{3}$$

1.4.4 Indifference curve and budget constraint



Chapter 2

Step by Step: Demand Functions, Substitution Effect and Income Effect

2.1 Setup

A consumer chooses a bundle (x, y) of two goods. Prices are p_x and p_y and income is I . The budget constraint is:

$$p_x x + p_y y = I$$

The demand function for each good is the optimal quantity of that good as a function of prices and income:

$$x(p_x, p_y, I) \quad y(p_x, p_y, I)$$

Three preferences:

$$u = x^{1/3}y^{2/3} \quad u = \frac{1}{3}x + \frac{2}{3}y \quad u = \min \left\{ \frac{1}{3}x, \frac{2}{3}y \right\}$$

2.2 Demand functions. Cobb-Douglas: $u = x^{1/3}y^{2/3}$

2.2.1 Tangency condition

Marginal utilities:

$$MU_x = \frac{1}{3}x^{-2/3}y^{2/3} \quad MU_y = \frac{2}{3}x^{1/3}y^{-1/3}$$

Dividing:

$$MRS = \frac{MU_x}{MU_y} = \frac{y}{2x}$$

Setting equal to p_x/p_y :

$$\frac{y}{2x} = \frac{p_x}{p_y} \implies p_y y = 2 p_x x$$

2.2.2 Demand functions

Substituting $p_y y = 2 p_x x$ into the budget constraint:

$$p_x x + 2 p_x x = I$$

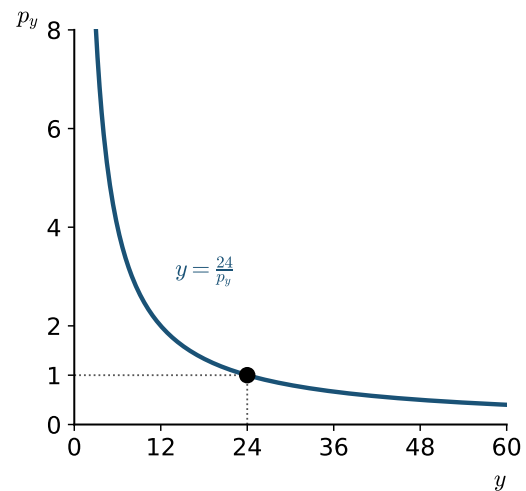
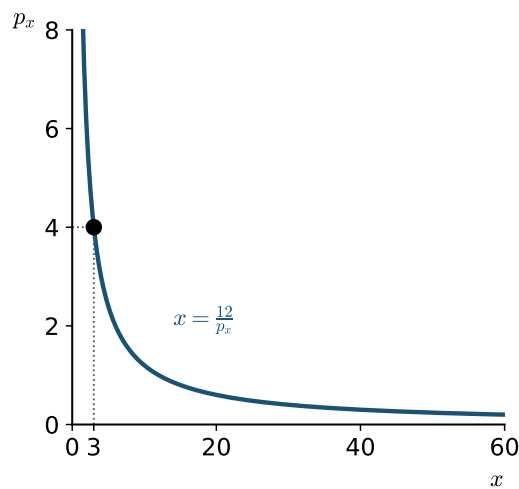
Solving:

$$x(p_x, p_y, I) = \frac{I}{3 p_x} \quad y(p_x, p_y, I) = \frac{2I}{3 p_y}$$

2.2.3 Demand curves

With $I = 36$:

$$x = \frac{36}{3 p_x} = \frac{12}{p_x} \quad y = \frac{2 \cdot 36}{3 p_y} = \frac{24}{p_y}$$



2.3 Demand functions. Perfect substitutes: $u = \frac{1}{3}x + \frac{2}{3}y$

2.3.1 Comparing MRS and p_x/p_y

Marginal utilities:

$$MU_x = \frac{1}{3} \quad MU_y = \frac{2}{3}$$

Dividing:

$$MRS = \frac{1/3}{2/3} = \frac{1}{2}$$

Comparing with p_x/p_y :

$$\frac{p_x}{p_y} > \frac{1}{2} \implies x = 0 \qquad \frac{p_x}{p_y} < \frac{1}{2} \implies y = 0$$

2.3.2 Demand functions

Substituting into the budget constraint:

$$x(p_x, p_y, I) = \begin{cases} 0 & \frac{p_x}{p_y} > \frac{1}{2} \\ \frac{I}{p_x} & \frac{p_x}{p_y} < \frac{1}{2} \end{cases} \qquad y(p_x, p_y, I) = \begin{cases} \frac{I}{p_y} & \frac{p_x}{p_y} > \frac{1}{2} \\ 0 & \frac{p_x}{p_y} < \frac{1}{2} \end{cases}$$

If $\frac{p_x}{p_y} = \frac{1}{2}$, any bundle on the budget constraint is optimal.

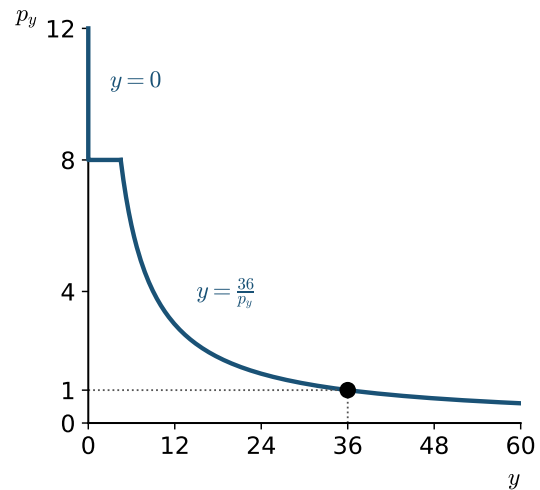
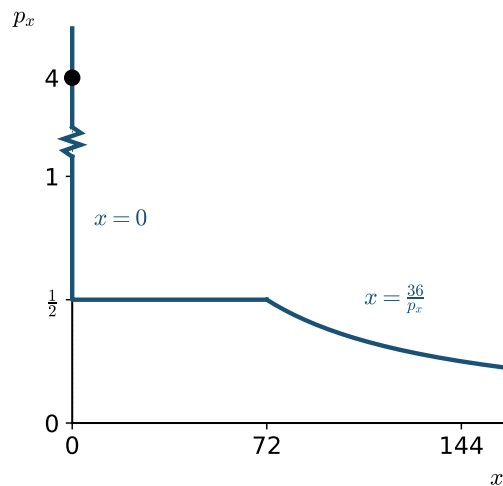
2.3.3 Demand curves

With $I = 36$ and $p_y = 1$:

$$x = \begin{cases} 0 & p_x > \frac{1}{2} \\ \frac{36}{p_x} & p_x < \frac{1}{2} \end{cases}$$

With $I = 36$ and $p_x = 4$:

$$y = \begin{cases} \frac{36}{p_y} & p_y < 8 \\ 0 & p_y > 8 \end{cases}$$



2.4 Demand functions. Perfect complements: $u = \min \left\{ \frac{1}{3}x, \frac{2}{3}y \right\}$

2.4.1 Fixed-proportions condition

Without wasting either good:

$$\frac{1}{3}x = \frac{2}{3}y \implies x = 2y$$

2.4.2 Demand functions

Substituting $x = 2y$ into the budget constraint:

$$p_x \cdot 2y + p_y y = I$$

Solving:

$$y(p_x, p_y, I) = \frac{I}{2p_x + p_y} \quad x(p_x, p_y, I) = \frac{2I}{2p_x + p_y}$$

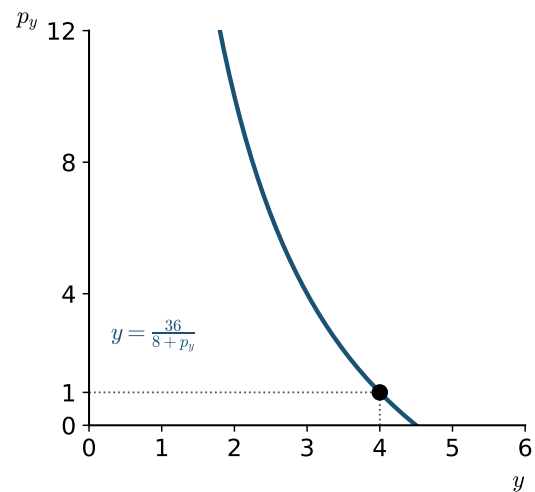
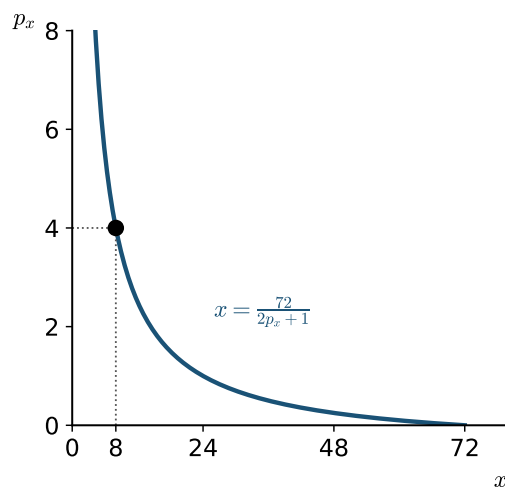
2.4.3 Demand curves

With $I = 36$ and $p_y = 1$:

$$x = \frac{72}{2p_x + 1}$$

With $I = 36$ and $p_x = 4$:

$$y = \frac{36}{8 + p_y}$$



2.5 Change in the price of x

The price of x falls. Income and the price of y do not change:

$$p_x = 4 \longrightarrow p'_x = \frac{1}{4} \quad p_y = 1 \quad I = 36$$

Three bundles:

- A : optimal bundle at the initial price, with utility U_1 .
- B : optimal bundle at the final price, with utility U_2 .
- A' : optimal bundle at the final price that keeps the initial utility U_1 .

Three budget constraints: BC_1 (initial price), BC_2 (final price) and BC' (parallel to BC_2 , through A').

Effects on the quantity of x :

$$SE = x_{A'} - x_A$$

$$IE = x_B - x_{A'}$$

$$TE = x_B - x_A = SE + IE$$

2.6 Substitution and income effects. Cobb-Douglas: $u = x^{1/3}y^{2/3}$

2.6.1 Initial bundle A

Evaluating the demand functions at $p_x = 4$:

$$x_A = \frac{36}{3 \cdot 4} = 3 \quad y_A = \frac{2 \cdot 36}{3 \cdot 1} = 24$$

Initial utility:

$$U_1 = 3^{1/3} \cdot 24^{2/3} = 12$$

2.6.2 Final bundle B

Evaluating the demand functions at $p'_x = \frac{1}{4}$:

$$x_B = \frac{36}{3 \cdot \frac{1}{4}} = 48 \quad y_B = \frac{2 \cdot 36}{3 \cdot 1} = 24$$

2.6.3 Compensated bundle A'

Tangency of the indifference curve through A with the final relative prices:

$$\frac{y}{2x} = \frac{1/4}{1} \implies y = \frac{1}{2}x$$

Substituting into $x^{1/3}y^{2/3} = U_1 = 12$:

$$x^{1/3} \left(\frac{1}{2}x\right)^{2/3} = 12 \implies \frac{x}{2^{2/3}} = 12$$

Solving:

$$x_{A'} = 12 \cdot 2^{2/3} \approx 19.05 \qquad y_{A'} = \frac{1}{2} x_{A'} = 6 \cdot 2^{2/3} \approx 9.52$$

Income of BC' :

$$\frac{1}{4} x_{A'} + y_{A'} = 3 \cdot 2^{2/3} + 6 \cdot 2^{2/3} = 9 \cdot 2^{2/3} \approx 14.29$$

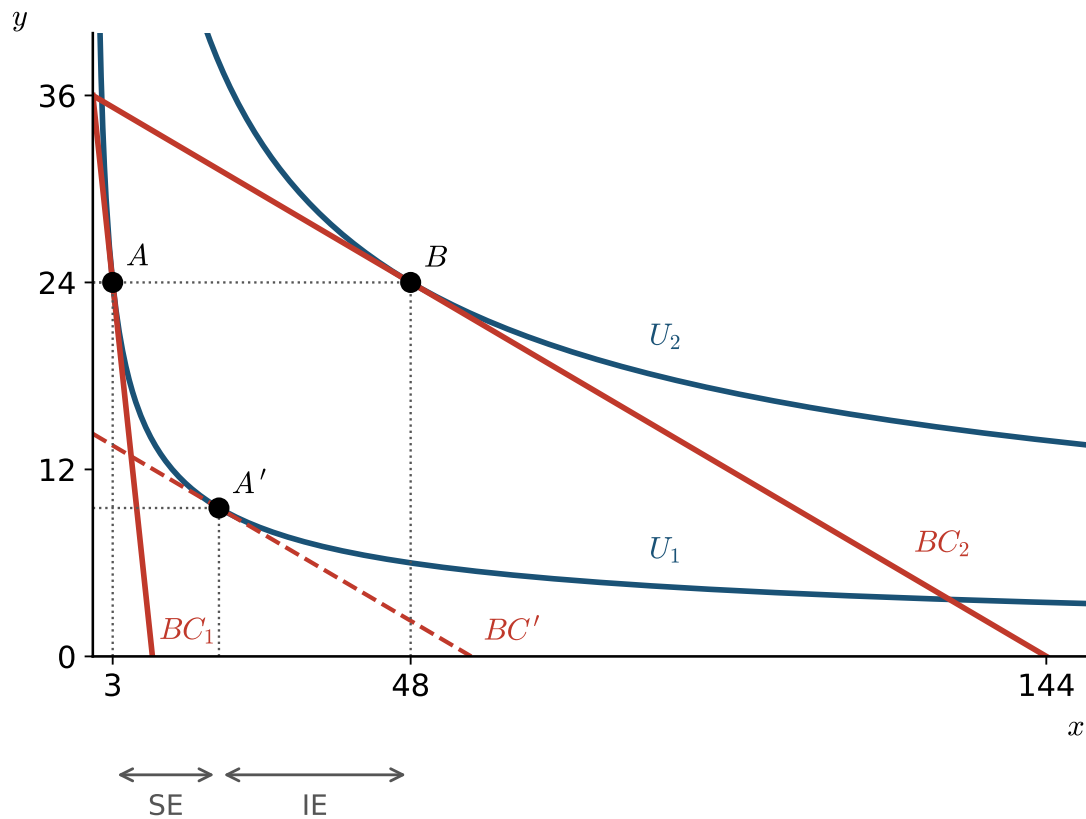
2.6.4 Substitution effect and income effect

$$SE = x_{A'} - x_A = 12 \cdot 2^{2/3} - 3 \approx 16.05$$

$$IE = x_B - x_{A'} = 48 - 12 \cdot 2^{2/3} \approx 28.95$$

$$TE = x_B - x_A = 48 - 3 = 45$$

2.6.5 Indifference curves and budget constraints



2.7 Substitution and income effects. Perfect substitutes: $u = \frac{1}{3}x + \frac{2}{3}y$

2.7.1 Initial bundle A

With $p_x = 4$:

$$\frac{p_x}{p_y} = 4 > \frac{1}{2} \implies x_A = 0 \quad y_A = \frac{36}{1} = 36$$

Initial utility:

$$U_1 = \frac{1}{3} \cdot 0 + \frac{2}{3} \cdot 36 = 24$$

2.7.2 Final bundle B

With $p'_x = \frac{1}{4}$:

$$\frac{p'_x}{p_y} = \frac{1}{4} < \frac{1}{2} \implies x_B = \frac{36}{1/4} = 144 \quad y_B = 0$$

2.7.3 Compensated bundle A'

At the final price, $\frac{p'_x}{p_y} < \frac{1}{2}$:

$$y_{A'} = 0$$

Substituting into $\frac{1}{3}x + \frac{2}{3}y = U_1 = 24$:

$$\frac{1}{3}x = 24 \implies x_{A'} = 72$$

Income of BC' :

$$\frac{1}{4} \cdot 72 + 0 = 18$$

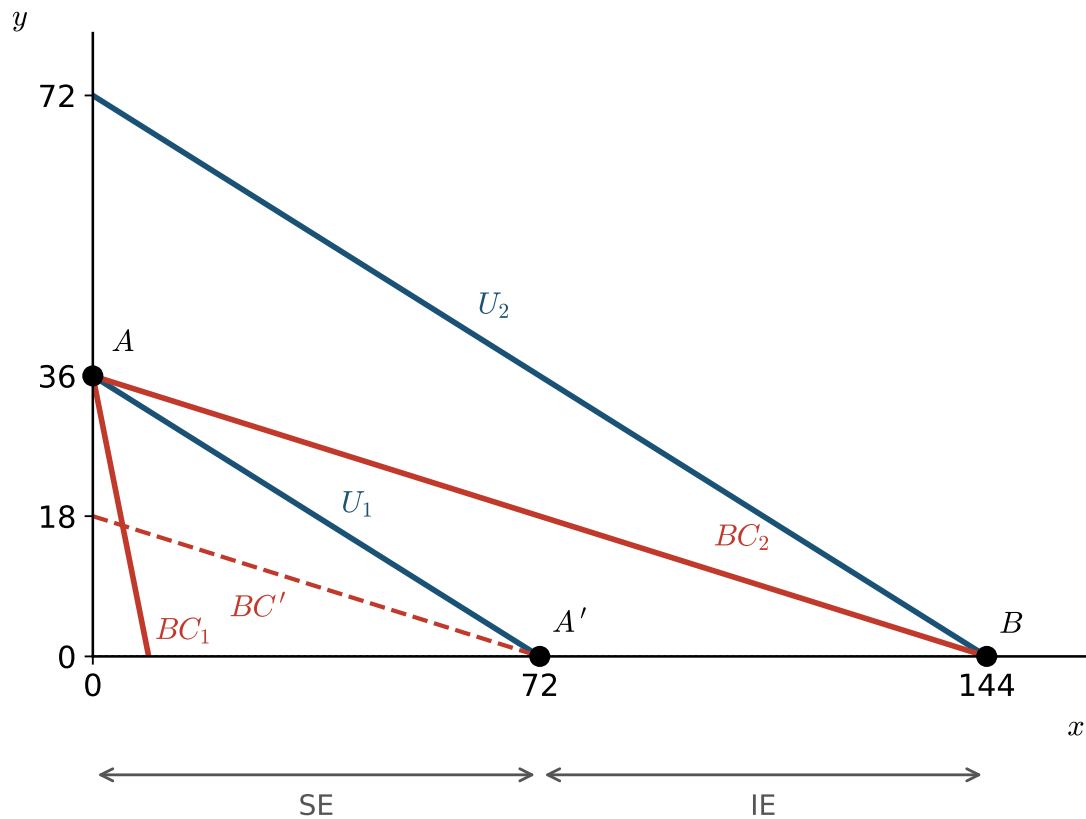
2.7.4 Substitution effect and income effect

$$SE = x_{A'} - x_A = 72 - 0 = 72$$

$$IE = x_B - x_{A'} = 144 - 72 = 72$$

$$TE = x_B - x_A = 144 - 0 = 144$$

2.7.5 Indifference curves and budget constraints



2.8 Substitution and income effects. Perfect complements: $u = \min \left\{ \frac{1}{3}x, \frac{2}{3}y \right\}$

2.8.1 Initial bundle A

Evaluating the demand functions at $p_x = 4$:

$$y_A = \frac{36}{2 \cdot 4 + 1} = 4 \quad x_A = \frac{2 \cdot 36}{2 \cdot 4 + 1} = 8$$

Initial utility:

$$U_1 = \min \left\{ \frac{1}{3} \cdot 8, \frac{2}{3} \cdot 4 \right\} = \frac{8}{3}$$

2.8.2 Final bundle B

Evaluating the demand functions at $p'_x = \frac{1}{4}$:

$$y_B = \frac{36}{2 \cdot \frac{1}{4} + 1} = 24 \quad x_B = \frac{2 \cdot 36}{2 \cdot \frac{1}{4} + 1} = 48$$

2.8.3 Compensated bundle A'

Without wasting either good, with utility $U_1 = \frac{8}{3}$:

$$\frac{1}{3}x = \frac{2}{3}y = \frac{8}{3} \implies x_{A'} = 8 \quad y_{A'} = 4$$

Therefore:

$$A' = A$$

Income of BC' :

$$\frac{1}{4} \cdot 8 + 4 = 6$$

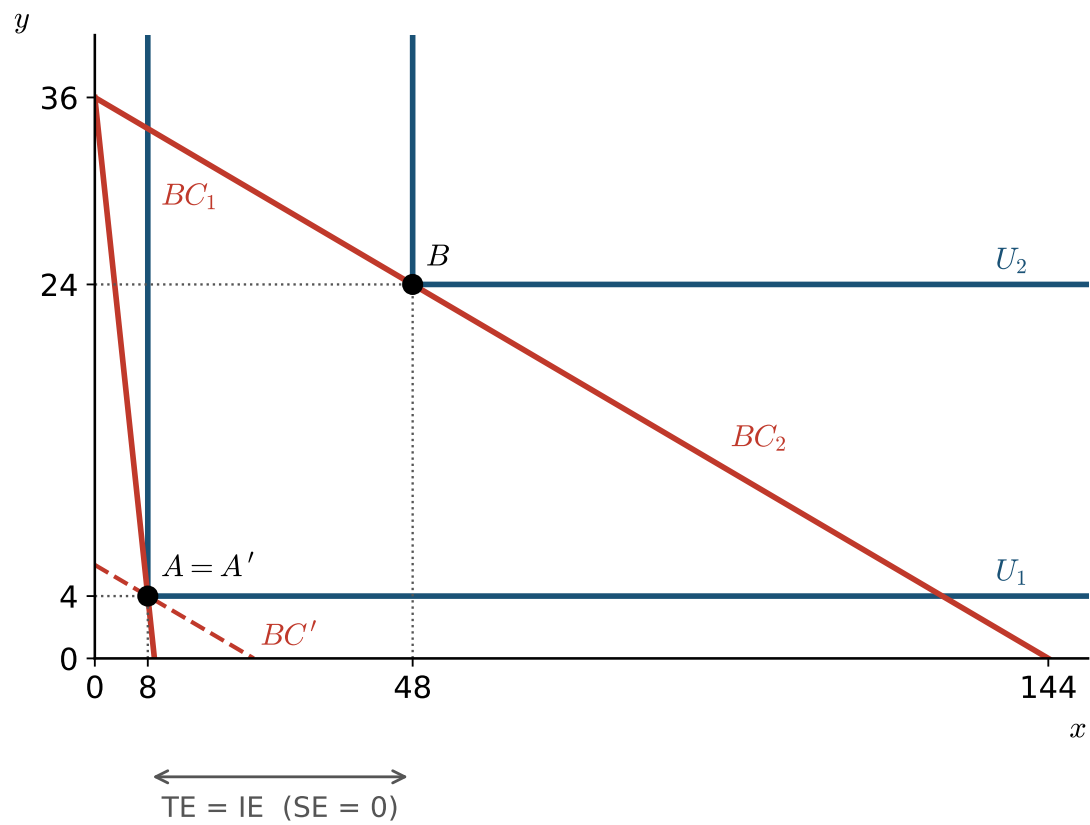
2.8.4 Substitution effect and income effect

$$SE = x_{A'} - x_A = 8 - 8 = 0$$

$$IE = x_B - x_{A'} = 48 - 8 = 40$$

$$TE = x_B - x_A = 48 - 8 = 40$$

2.8.5 Indifference curves and budget constraints



Chapter 3

Step by Step: Short-Run and Long-Run Cost Functions

3.1 Setup

A firm produces q units of output using labor, L , and capital, K :

$$q = f(L, K)$$

The price of labor is w and the price of capital is r :

$$w = 4 \qquad r = 1$$

The cost of producing with (L, K) is:

$$TC = wL + rK$$

In the long run (lr) the firm chooses both L and K . In the short run (sr) capital is fixed at $\bar{K}$ and the firm chooses only L :

$$\bar{K} = 8$$

Three technologies:

$$q = L^{1/3} K^{2/3} \qquad q = \frac{1}{3}L + \frac{2}{3}K \qquad q = \min \left\{ \frac{1}{3}L, \frac{2}{3}K \right\}$$

3.2 Cobb-Douglas: $q = L^{1/3} K^{2/3}$

3.2.1 Short run: conditional labor demand

With $\bar{K} = 8$:

$$\bar{K}^{2/3} = 8^{2/3} = 4$$

Substituting into the production function:

$$q = L^{1/3} \bar{K}^{2/3} = 4 L^{1/3}$$

Solving for L :

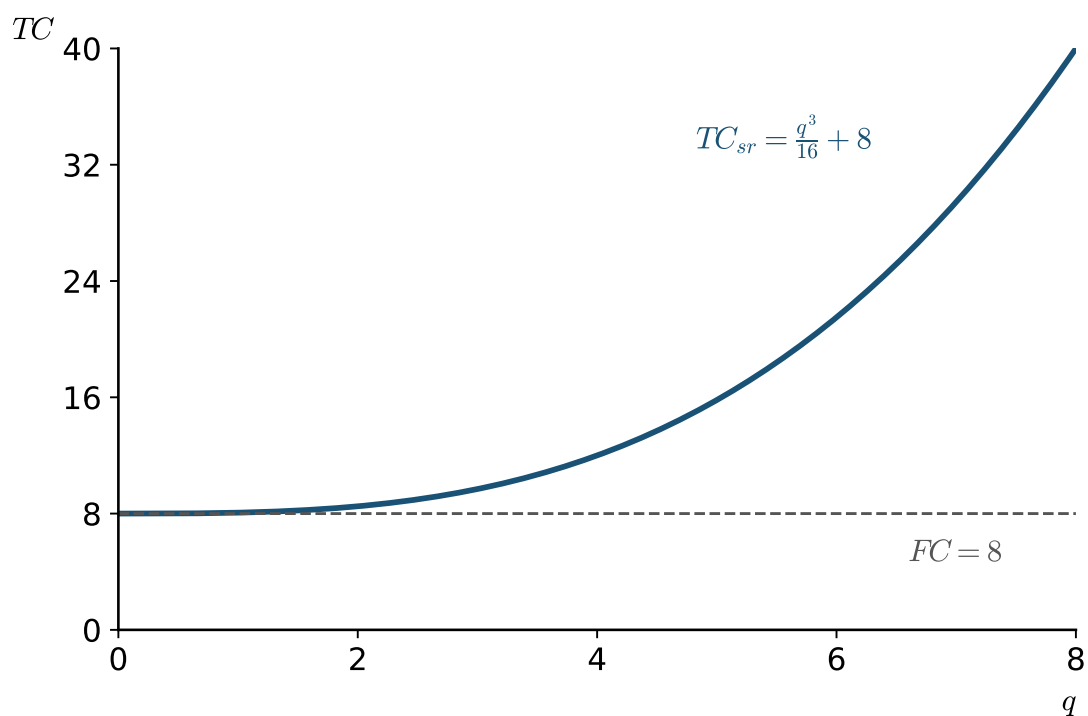
$$L^{1/3} = \frac{q}{4} \implies L(q) = \frac{q^3}{64}$$

3.2.2 Short run: cost function

$$TC_{sr}(q) = w L(q) + r \bar{K} = 4 \cdot \frac{q^3}{64} + 1 \cdot 8$$

Simplifying:

$$TC_{sr}(q) = \underbrace{\frac{q^3}{16}}_{VC} + \underbrace{8}_{FC}$$



3.2.3 Short run: marginal cost and average cost

Differentiating TC_{sr} :

$$MC_{sr}(q) = \frac{dTC_{sr}}{dq} = \frac{3q^2}{16}$$

Dividing TC_{sr} by q :

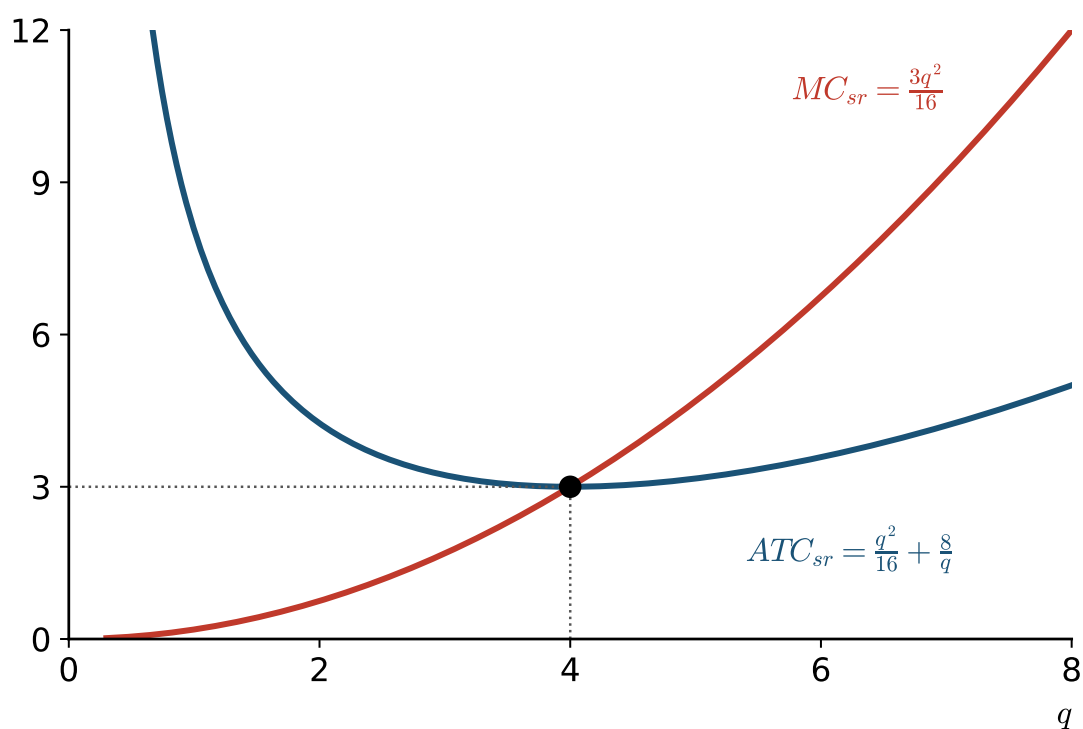
$$ATC_{sr}(q) = \frac{TC_{sr}}{q} = \frac{q^2}{16} + \frac{8}{q}$$

Minimum of ATC_{sr} :

$$\frac{dATC_{sr}}{dq} = \frac{q}{8} - \frac{8}{q^2} = 0 \implies q^3 = 64 \implies q = 4$$

Evaluating at $q = 4$:

$$ATC_{sr}(4) = 1 + 2 = 3 \qquad MC_{sr}(4) = \frac{3 \cdot 16}{16} = 3$$



3.2.4 Long run: tangency condition

Marginal products:

$$MP_L = \frac{1}{3}L^{-2/3}K^{2/3} \qquad MP_K = \frac{2}{3}L^{1/3}K^{-1/3}$$

Dividing:

$$MRTS = \frac{MP_L}{MP_K} = \frac{K}{2L}$$

Setting it equal to w/r :

$$\frac{K}{2L} = \frac{4}{1} \implies K = 8L$$

3.2.5 Long run: conditional factor demands

Substituting $K = 8L$ into the production function:

$$q = L^{1/3}(8L)^{2/3} = 4L$$

Solving:

$$L(q) = \frac{q}{4} \quad K(q) = 8L(q) = 2q$$

3.2.6 Long run: cost function

$$TC_{lr}(q) = wL(q) + rK(q) = 4 \cdot \frac{q}{4} + 1 \cdot 2q$$

Simplifying:

$$TC_{lr}(q) = 3q$$

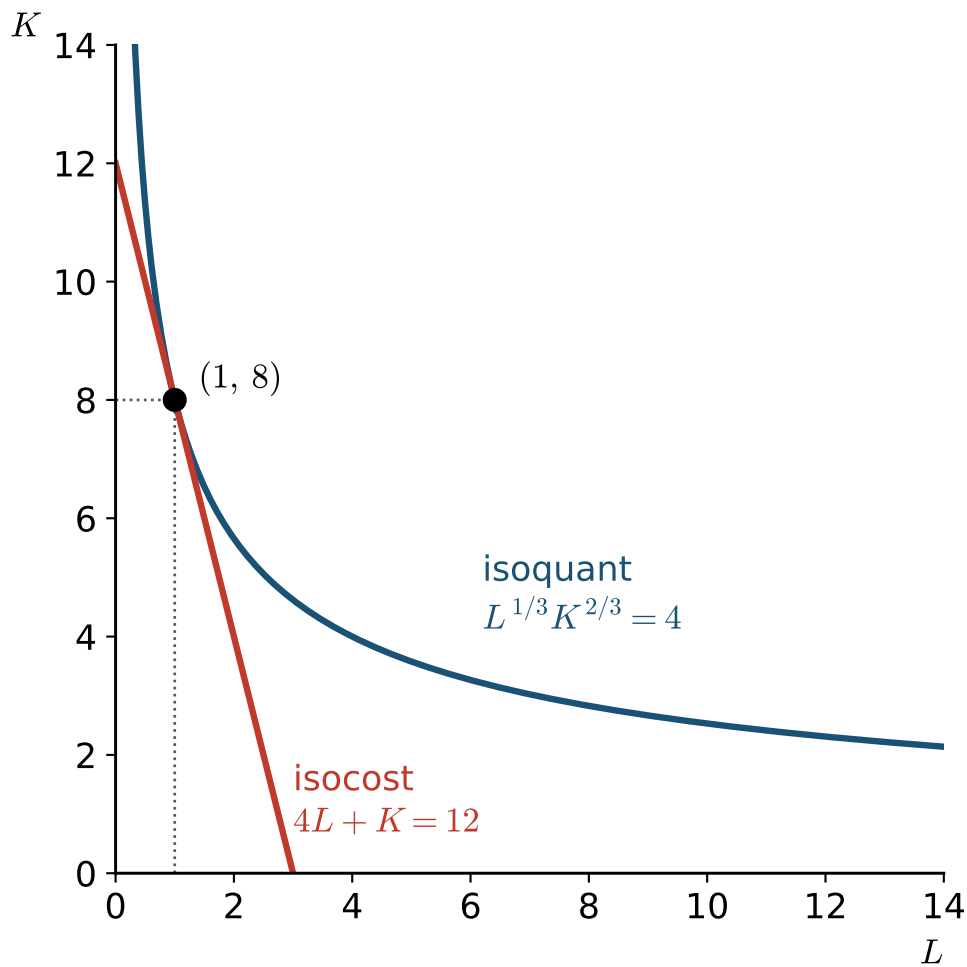
Marginal cost and average cost:

$$MC_{lr}(q) = 3 \quad ATC_{lr}(q) = 3$$

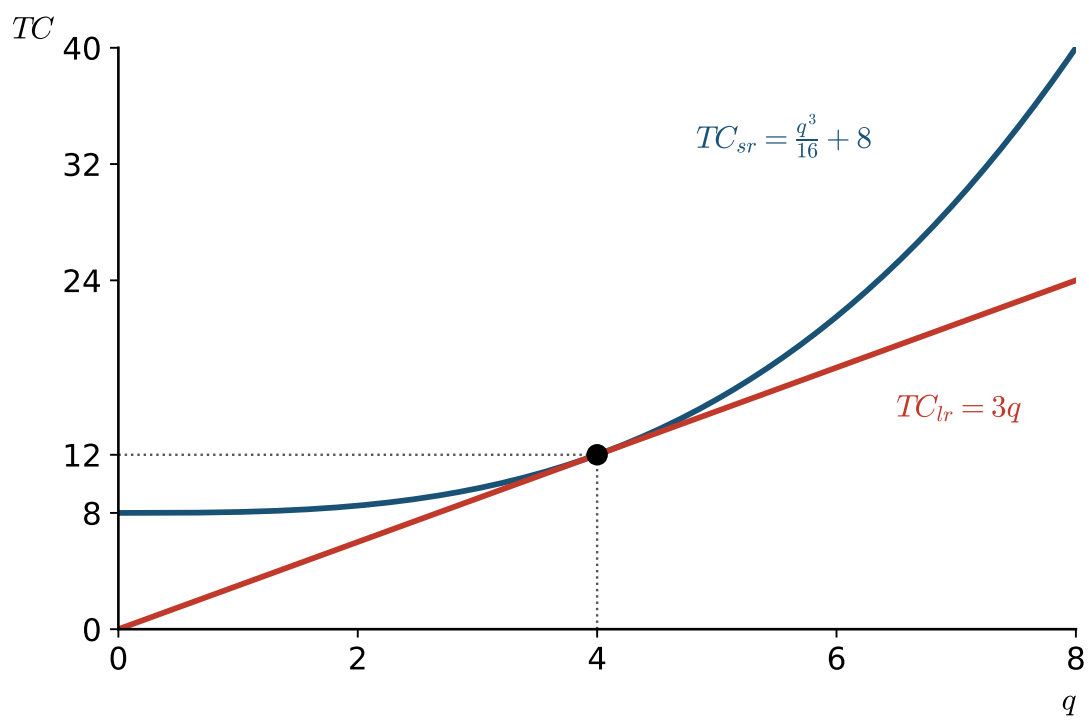
3.2.7 Isoquant and tangent isocost

For $q = 4$:

$$L(4) = 1 \quad K(4) = 8 \quad TC_{lr}(4) = 12$$



3.2.8 Short-run and long-run cost functions



3.2.9 Returns to scale

$$f(tL, tK) = (tL)^{1/3}(tK)^{2/3} = t^{1/3+2/3} L^{1/3} K^{2/3}$$

Therefore:

$$f(tL, tK) = t f(L, K)$$

The technology exhibits constant returns to scale.

3.3 Perfect substitutes: $q = \frac{1}{3}L + \frac{2}{3}K$

3.3.1 Short run: conditional labor demand

With $\bar{K} = 8$:

$$q = \frac{1}{3}L + \frac{2}{3}\bar{K} = \frac{1}{3}L + \frac{16}{3}$$

Solving for L :

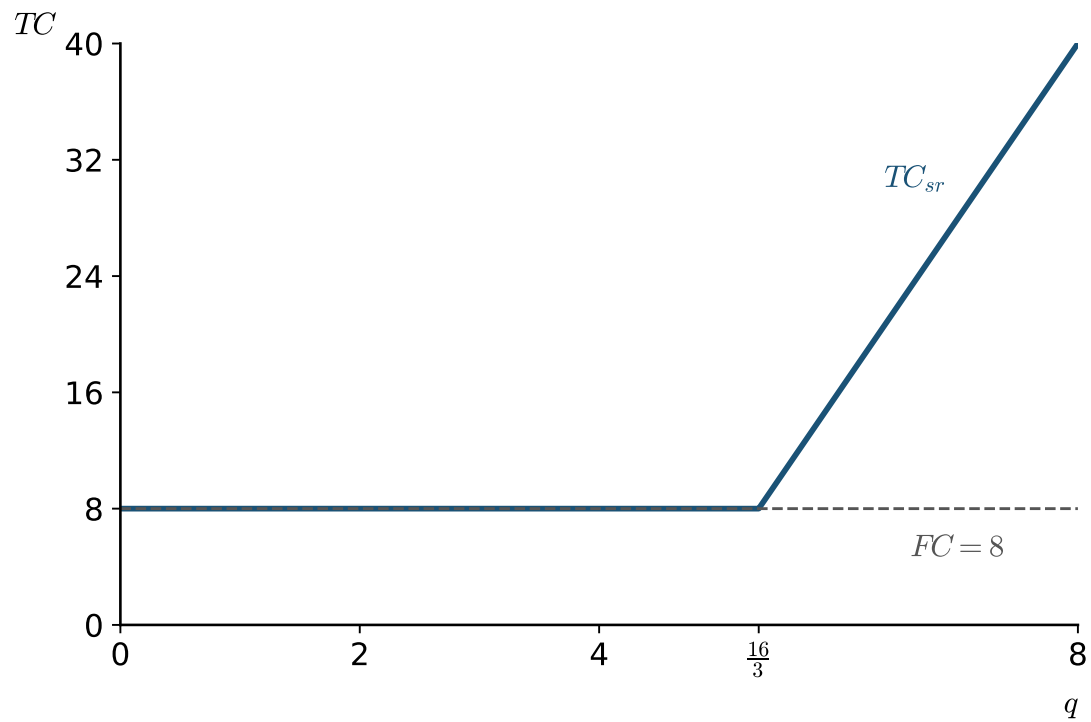
$$L(q) = \begin{cases} 0 & q \leq \frac{16}{3} \\ 3q - 16 & q > \frac{16}{3} \end{cases}$$

3.3.2 Short run: cost function

$$TC_{sr}(q) = w L(q) + r \bar{K} = 4 L(q) + 1 \cdot 8$$

Substituting $L(q)$:

$$TC_{sr}(q) = \begin{cases} 8 & q \leq \frac{16}{3} \\ 12q - 56 & q > \frac{16}{3} \end{cases}$$



3.3.3 Short run: marginal cost and average cost

Differentiating TC_{sr} :

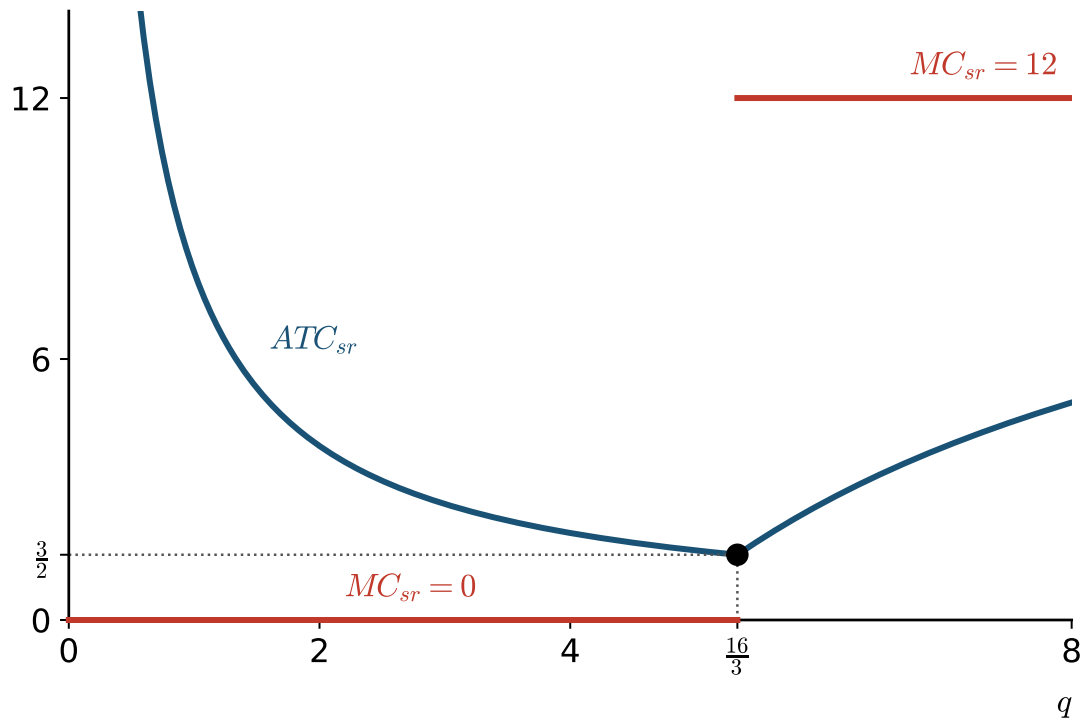
$$MC_{sr}(q) = \begin{cases} 0 & q < \frac{16}{3} \\ 12 & q > \frac{16}{3} \end{cases}$$

Dividing TC_{sr} by q :

$$ATC_{sr}(q) = \begin{cases} \frac{8}{q} & q \leq \frac{16}{3} \\ 12 - \frac{56}{q} & q > \frac{16}{3} \end{cases}$$

Minimum of ATC_{sr} , at $q = \frac{16}{3}$:

$$ATC_{sr}\left(\frac{16}{3}\right) = \frac{8}{16/3} = \frac{3}{2}$$



3.3.4 Long run: comparing $MRTS$ and w/r

Marginal products:

$$MP_L = \frac{1}{3} \quad MP_K = \frac{2}{3}$$

Dividing:

$$MRTS = \frac{1/3}{2/3} = \frac{1}{2} \quad \frac{w}{r} = 4$$

Comparing:

$$MRTS < \frac{w}{r} \implies L = 0$$

3.3.5 Long run: conditional factor demands

Substituting $L = 0$ into the production function:

$$q = \frac{2}{3}K$$

Solving:

$$L(q) = 0 \quad K(q) = \frac{3}{2}q$$

3.3.6 Long run: cost function

$$TC_{lr}(q) = w L(q) + r K(q) = 4 \cdot 0 + 1 \cdot \frac{3}{2}q$$

Simplifying:

$$TC_{lr}(q) = \frac{3}{2}q$$

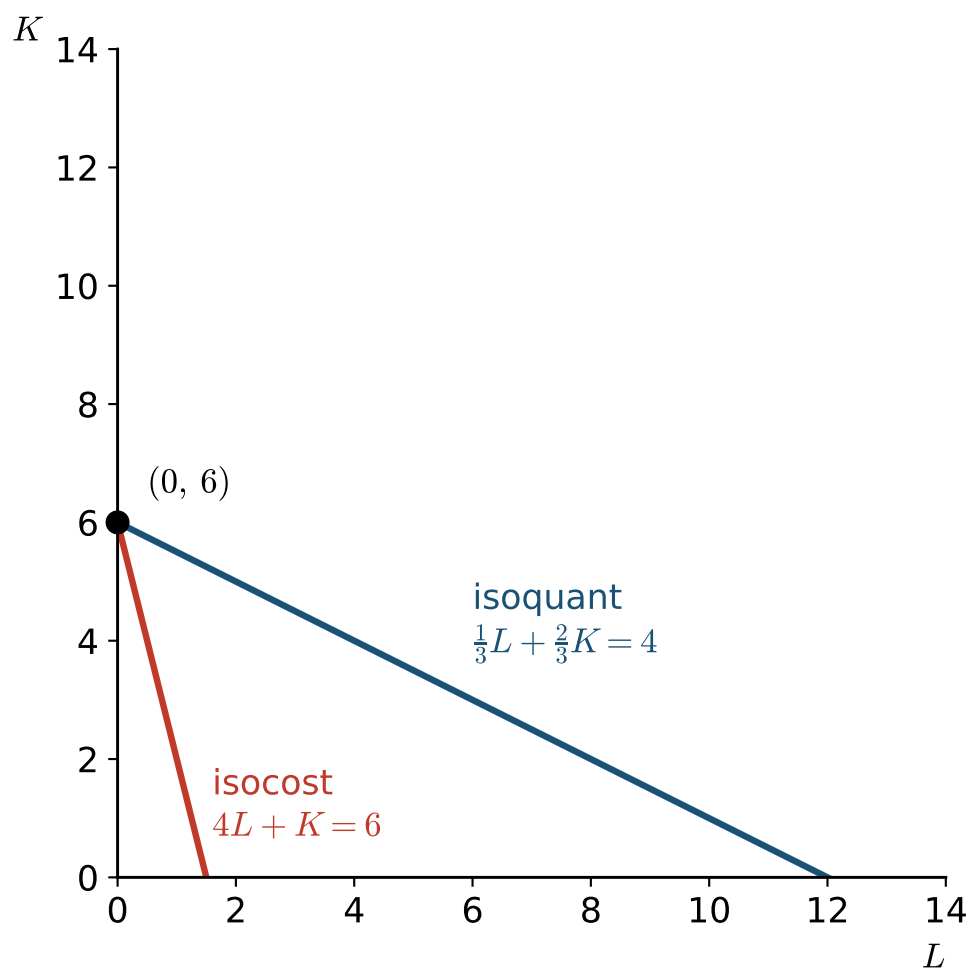
Marginal cost and average cost:

$$MC_{lr}(q) = \frac{3}{2} \quad ATC_{lr}(q) = \frac{3}{2}$$

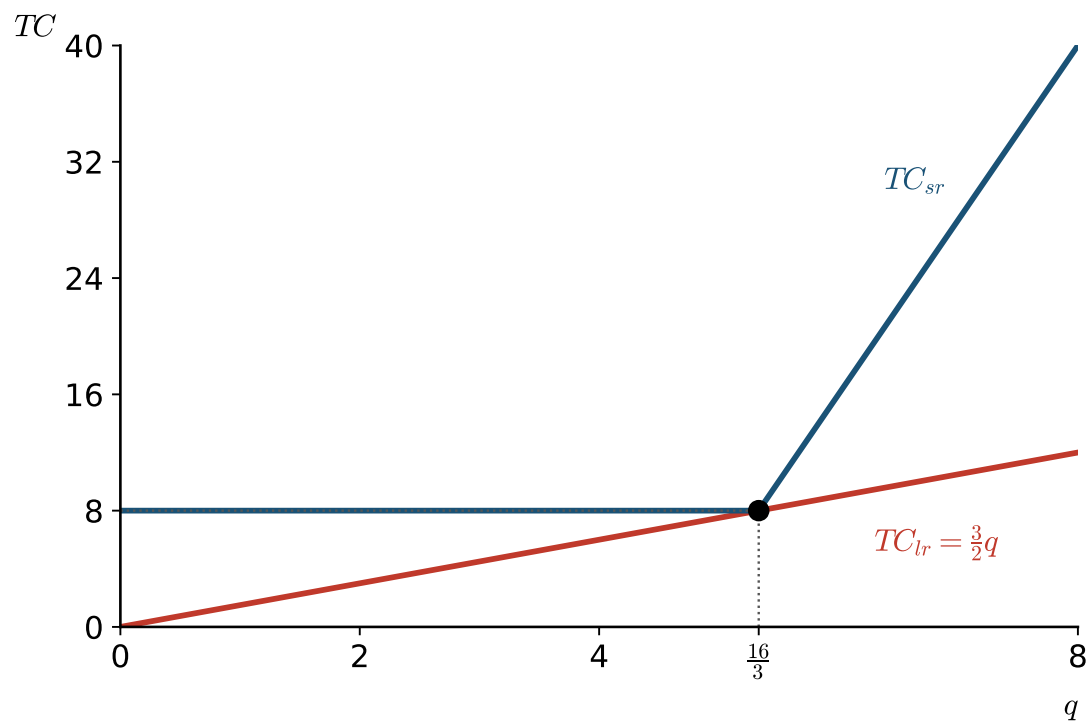
3.3.7 Isoquant and isocost

For $q = 4$:

$$L(4) = 0 \quad K(4) = 6 \quad TC_{lr}(4) = 6$$



3.3.8 Short-run and long-run cost functions



3.3.9 Returns to scale

$$f(tL, tK) = \frac{1}{3}(tL) + \frac{2}{3}(tK) = t \left(\frac{1}{3}L + \frac{2}{3}K \right)$$

Therefore:

$$f(tL, tK) = t f(L, K)$$

The technology exhibits constant returns to scale.

3.4 Perfect complements: $q = \min \left\{ \frac{1}{3}L, \frac{2}{3}K \right\}$

3.4.1 Short run: conditional labor demand

With $\bar{K} = 8$:

$$q = \min \left\{ \frac{1}{3}L, \frac{2}{3}\bar{K} \right\} = \min \left\{ \frac{1}{3}L, \frac{16}{3} \right\} \implies q \leq \frac{16}{3}$$

Solving for L :

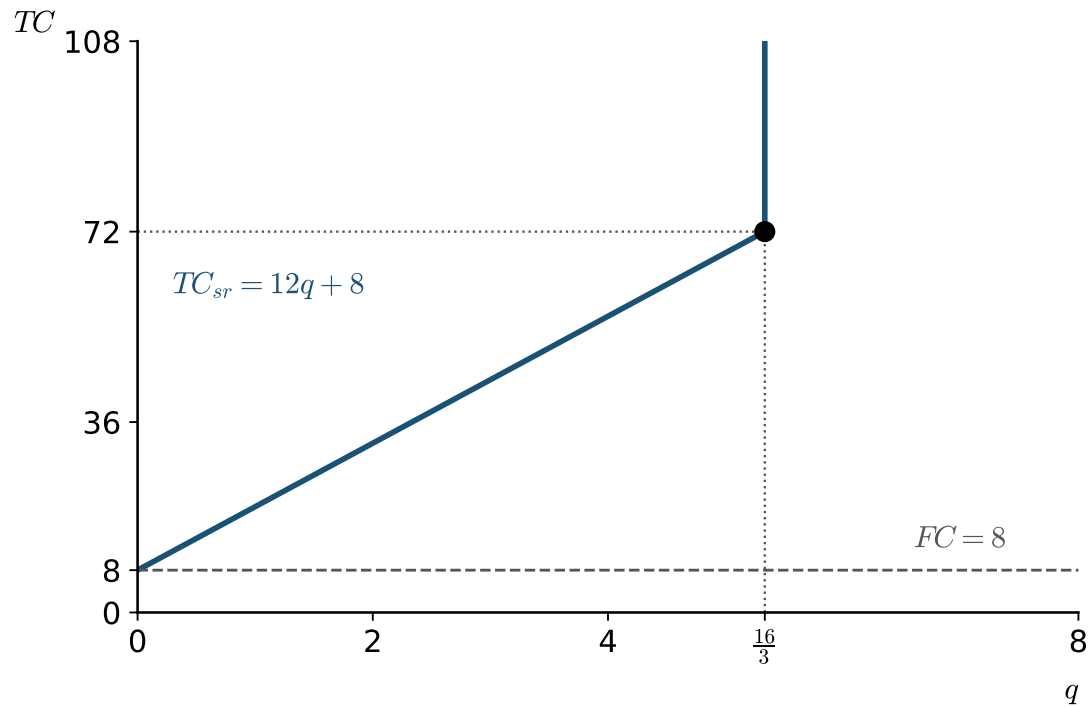
$$L(q) = 3q \quad \left(q \leq \frac{16}{3} \right)$$

3.4.2 Short run: cost function

$$TC_{sr}(q) = wL(q) + r\bar{K} = 4 \cdot 3q + 1 \cdot 8$$

Simplifying:

$$TC_{sr}(q) = \underbrace{12q}_{\widetilde{VC}} + \underbrace{8}_{\widetilde{FC}} \quad \left(q \leq \frac{16}{3}\right)$$



3.4.3 Short run: marginal cost and average cost

Differentiating TC_{sr} :

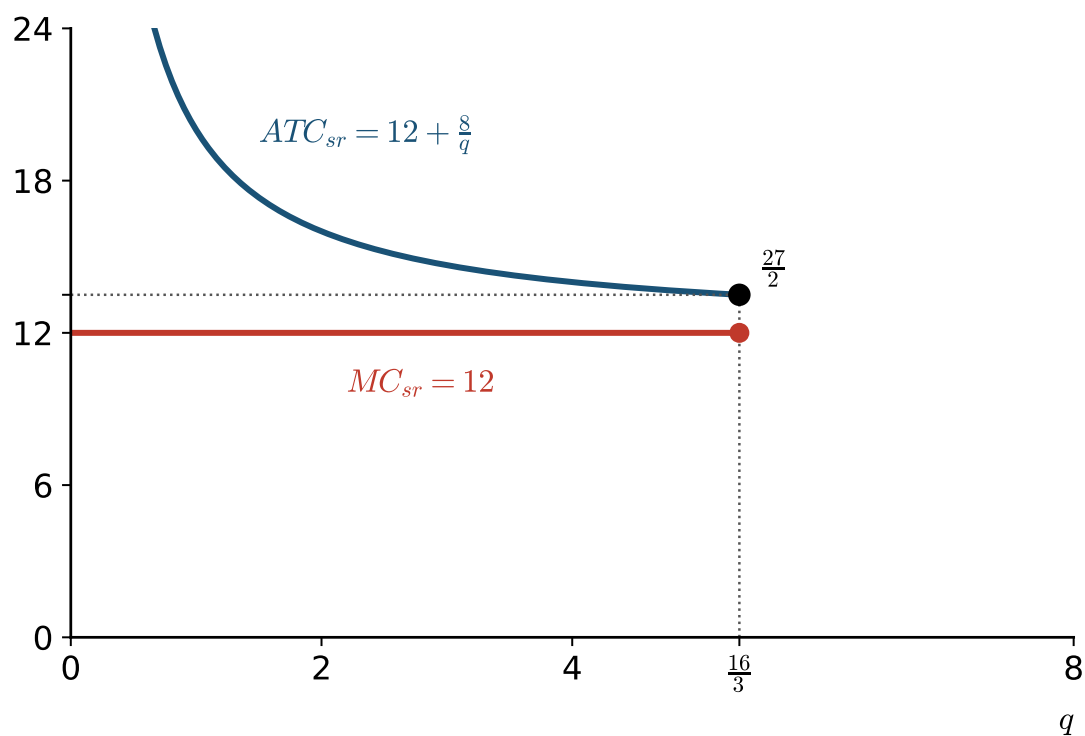
$$MC_{sr}(q) = 12 \quad \left(q \leq \frac{16}{3}\right)$$

Dividing TC_{sr} by q :

$$ATC_{sr}(q) = 12 + \frac{8}{q} \quad \left(q \leq \frac{16}{3}\right)$$

Minimum of ATC_{sr} , at $q = \frac{16}{3}$:

$$ATC_{sr}\left(\frac{16}{3}\right) = 12 + \frac{8}{16/3} = \frac{27}{2}$$



3.4.4 Long run: conditional factor demands

Wasting neither input:

$$\frac{1}{3}L = \frac{2}{3}K = q$$

Solving:

$$L(q) = 3q \quad K(q) = \frac{3}{2}q$$

3.4.5 Long run: cost function

$$TC_{lr}(q) = wL(q) + rK(q) = 4 \cdot 3q + 1 \cdot \frac{3}{2}q$$

Simplifying:

$$TC_{lr}(q) = \frac{27}{2}q$$

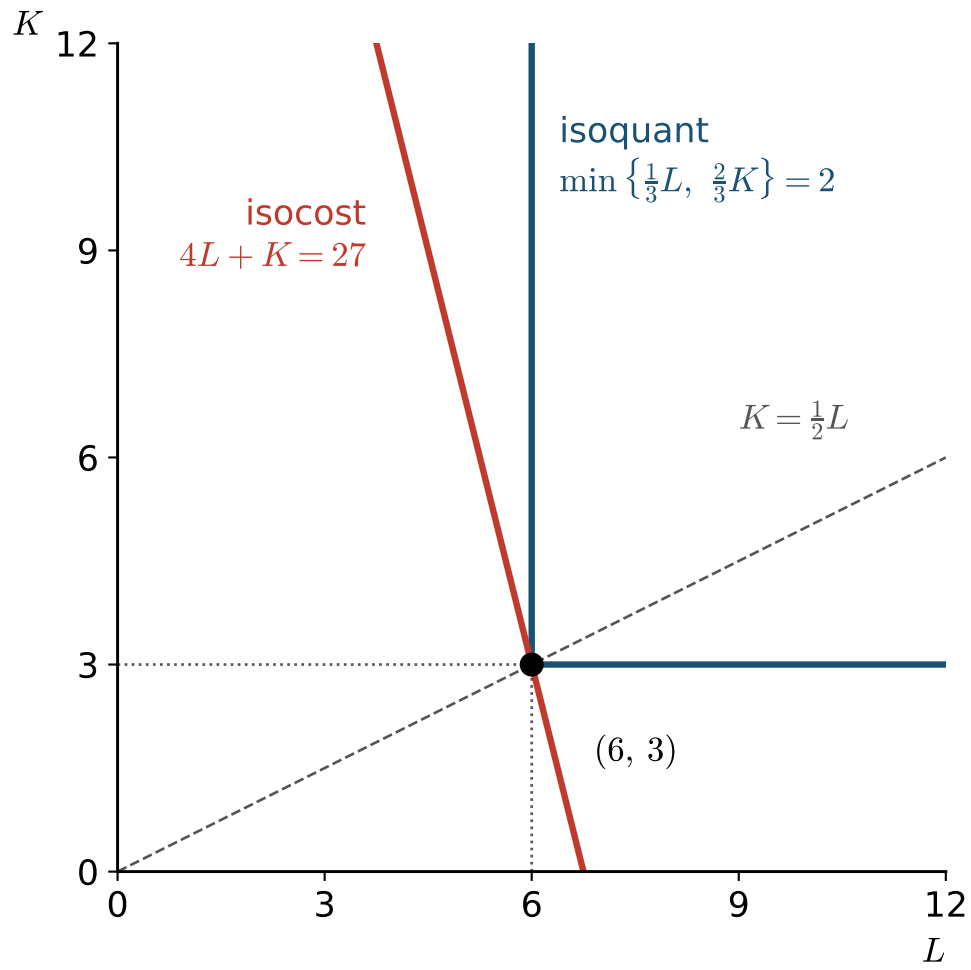
Marginal cost and average cost:

$$MC_{lr}(q) = \frac{27}{2} \quad ATC_{lr}(q) = \frac{27}{2}$$

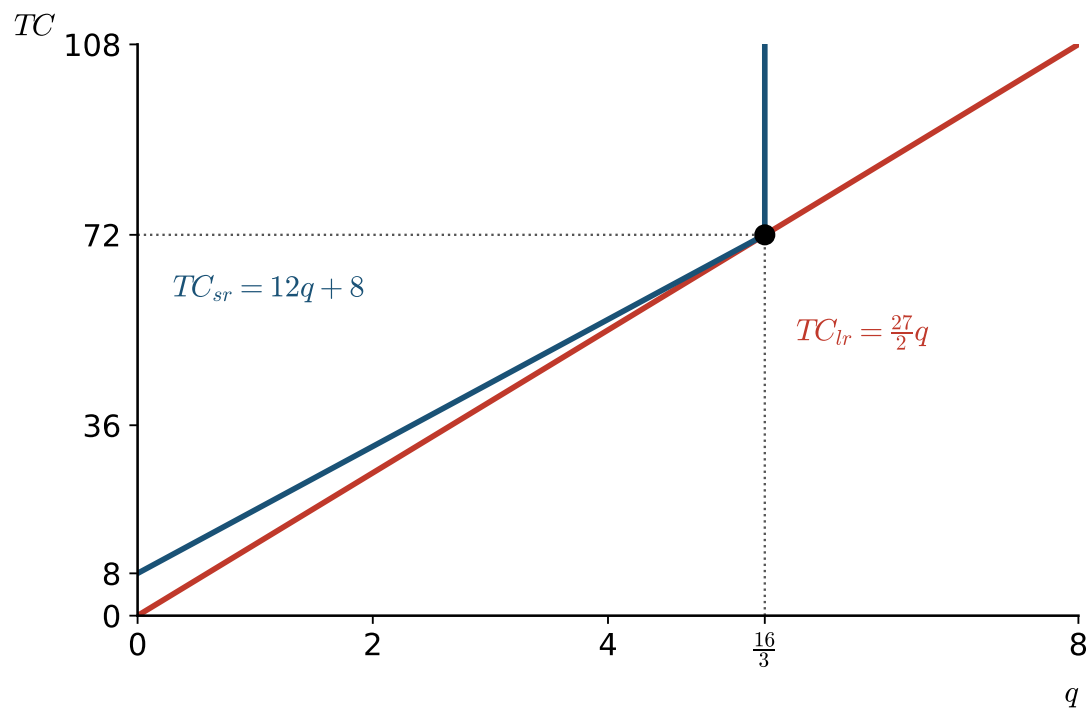
3.4.6 Isoquant and isocost

For $q = 2$:

$$L(2) = 6 \quad K(2) = 3 \quad TC_{lr}(2) = 27$$



3.4.7 Short-run and long-run cost functions



3.4.8 Returns to scale

$$f(tL, tK) = \min \left\{ \frac{1}{3}tL, \frac{2}{3}tK \right\} = t \min \left\{ \frac{1}{3}L, \frac{2}{3}K \right\}$$

Therefore:

$$f(tL, tK) = t f(L, K)$$

The technology exhibits constant returns to scale.