Public Economies and the Endogenous Choice of Institutions*

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Abstract

In this paper we provide a framework in which to formalize the seminal work of Elinor Ostrom on the study of public economies, a prominent theoretical construct aimed to provide answers to the following questions: (*i*) Why are some societies able to solve their collective action problems and others are not? and (*ii*) Why do societies choose the particular institutions they choose from a vast array of possible choices?

KEYWORDS: Public Economies, Institutional Design, Collective Action.

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1 Introduction

Why are some societies able to solve their collective action problems and others are not? Why do societies choose the particular institutions they choose from a vast array of possible choices? In this article we provide a framework in which to formalize the seminal work of the recent Nobel Laureate Elinor Ostrom on the study of public economies, a prominent theoretical construct aimed to provide answers to these important questions.

A public economy, simply defined, is a form of economic organization dedicated to the production, provision and consumption of collective goods. In several studies, Ostrom and her co-authors have developed a set of hypotheses regarding the process by which public economies have evolved to provide and produce public goods and maintain common pool resources throughout the world. This process, according to Ostrom, cannot be summarized by simply arguing that "the market" is the resulting solution to all the situations in which one can internalize at the individual level the (external) effects of the collective action problem at hand and that "the state" is the resulting solution to all problems in which this is not the case. Instead, Ostrom argues, the real world exhibits a variety of institutions, all of them imperfect in nature, by which those collective action problems are routinely solved. These include "families and clans, neighborhood associations, communal organizations, trade associations, buyers and producer's cooperatives, local voluntary associations and clubs, special districts, international regimes, public service industries, arbitration and mediation associations, and charitable organizations," [23, p. 36] among others.

Public economies are neither markets nor hierarchies and, according to Ostrom, they are not very well understood. It is Ostrom's opinion that there are two main open problems regarding the study of public economies. The first is that (i) we do not yet understand why some societies are able to solve their collective action problems and others are not. The second is that (ii) we do not yet understand why societies choose the particular institutions they choose from the vast array of possible choices. In Ostrom's own words: "How a group of principals—a community of citizens—can organize themselves to solve the problems of institutional supply, commitment, and monitoring is still a theoretical puzzle" [20, p. 29]. These are the questions we set out to answer in this paper in the context of a simple model of a public economy.

The organization of the remainder of the paper is as follows: In Section 2 we present some elements of Ostrom's research on the study of public economies. In Section 3 we develop a formalism, and a simple example of a public economy (involving two competing providers of local public goods and a population with heterogeneous valuation of those goods) in which we can investigate the answers to questions (i)

and (*ii*) above. In Section 4 we discuss the results. Section 5 concludes.

2 Public Economies

In this section we draw from Ostrom [21] and Ostrom and Walker [23], in which the main conceptual issues that surround the study of public economies are presented.

Public economies are forms of economic organization "composed of *collective consumption units* of varying sizes that provide services by arranging for their production and regulating access to, patterns of use, and appropriation of collective goods" [21, pp. 6-7].

There are two key components to this definition. The first component acknowledges the need to organize consumption through the creation of collective consumption units. Whenever exclusion is problematic—as with public goods and common-pool resources—"creating a collective consumption unit larger than a household is essential to overcome problems of free riding and strategic preference revelation, to determine how costs will be shared among those who benefit, to arrange for production, and to regulate patterns of access, use, and appropriation" [21, p. 7].

The second key component to the definition of a public economy is that the provision of services is viewed as a distinct process apart from production: "The primary reason for using a form of collective organization is to solve problems of provision. But once a collective consumption unit is established, how production is organized is an entirely *separate* question" [21, p. 7]. Therefore, the producers in a public economy may or may not be the same organizing unit as the collective consumption unit that organizes the provision side.

2.1 How to Study a Public Economy (I)

The canonical manner in which modern economists and political scientists study public economies is by using tools from the theory of non-cooperative games.[1] Those tools have been extremely influential in spreading the belief that phenomena like the "tragedy of the commons" are the resulting outcome from the process of provision and appropriation of collective goods *in the presence of extremely sparse institutional structures.*

While this belief is uncontested by the existing theoretical and empirical literatures on the topic, considerable disagreement exists regarding the extent to which individuals are passive recipients of such sparse institutional structures and the

[1] For a prominent exception see Moulin [18].

"tragedy" results unavoidable. According to Ostrom and Walker [23] these results are not the necessary outcome in a public economy precisely because individuals act to transform the rules of interaction into ones in which the "tragedy" can be avoided.

Kiser and Ostrom [14] distinguish two levels of rules that cumulatively affect the actions taken and the outcomes obtained in any setting:

1. *Operational rules* directly affecting day-to-day decisions made by the participants in any setting, and

2. *Constitutional and collective-choice rules* affecting operational activities and outcomes through their effects in determining who is eligible and the specific rules to be used in changing operational rules.

Importantly, "once one recognizes that those involved in collective action may shift out of a current 'game' to a deeper-level game, *the necessity of using multiple levels of analysis becomes apparent.* All rules are nested in another set of rules that if enforced defines how the first set of rules can be changed." [23, p. 43] (our italics) Despite this, "most of the emphasis in the public choice tradition has been on predicting behavior *within* the structure of a game, rather than on the processes of organizing new games and on self-monitoring and sanctioning activities." [23, p. 45] What is important for the analysis of public economies, however, is "to recognize that individuals can consciously decide to adopt their own rules that either replace or complement the rules governing an initial collective action situation." [23, p. 43]

2.2 Providing Explanations of the Rules of the Game

The key problem regarding the modern study of public economies is that we know little about how these rules that govern a collective action situation change over time. The point of view maintained by Ostrom and Walker [23] is that, while individual behavior reacts quickly once the rules that govern the situation are clear, the process of rule formation is rich in difficulties and uncertainties: "changes in deeper-level rules usually are more difficult and more costly to accomplish, thus increasing the stability of mutual expectations among individuals interacting according to a set of rules." [23, p. 43] This motivates Ostrom to assert that "given these levels of uncertainty about the basic structure of the problem appropriators face, the only reasonable assumption to make about the discovery and calculation processes employed [to find the best possible rules of the game] is that appropriators engage in a considerable amount of trial-and-error learning (...) By definition, trial-and-error methods involve errors, perhaps even disasters. Over time, appropriators gain a more accurate understanding of the physical world and what to expect from the behavior of others" [20, p. 34].[2] An important aspect of Ostrom's views on the outcomes of such a process of rule formation is that there is no compelling reason to expect necessarily (first-best) "optimal" rules, yet one should expect the resulting institutions to improve over time whenever the individuals involved have sufficient autonomy to craft their own institutions.

As researchers like Pierson [24] have noted, path dependence may also play a significant role in determining the institutions on which a society depends. He describes path dependence as such: "[O]nce a [society] has started down a track, the costs of reversal are very high...In [a path dependent] process, the probability of further steps along the same path increases with each move down that path. This is because the *relative* benefits of the current activity compared with other possible options increase over time." [24, p. 252] Although path dependence may affect the set of institutions from which a society can choose, we believe that our model provides insight into how societies select from among the institutions in their choice set, even if that choice occurs through trial and error over time. Pierson acknowledges that "path dependent analyses need not imply that a particular alternative is permanently locked in," [24, p. 265] rather, that a society's history makes particular institutions more or less likely to be adopted in the future. Thus, we believe our analysis provides explanatory power even in path dependent environments.

The relationship between the producers of collective goods and the collective consumption units is central to Ostrom's research. Specifically, her work considers three types of relationships between these parties:

1. The producers and consumers of collective goods can be the same group of individuals; this setting has been studied extensively in the household economics literature [1].

2. The producers may be a distinct party from the consumers, however, in some cases, the collective good—in order to be successfully consumed—must be "co-produced" by the consumption unit [21, p. 10]. For example, police services are provided by a distinct group of producers (police officers) for an external collective consumption unit (citizens). However, the degree to which the services provide benefits to the consumption unit depends on the participation of

[2]That rules of the game change over time in this fashion is of course not an idea that is arcane to social scientists. To wit: "the rules of the game (...) are all akin to equilibrium expectations; the product of long-term experience by a society of boundedly rational and retrospective individuals" Kreps [15, pp. 182-3]; "it would seem that an appropriate topic of a truly evolutionary game theory is evolution in the rules of the game" Blume [2, p. 31].

the members of the consumption unit themselves. As Ostrom explains, "To be effective, police officers need the active coproduction of citizens...Without active help by citizens giving information and being willing to serve as witnesses in court, police are less effective in solving crimes" [21, p. 10].

3. The producers of collective goods can be completely distinct from the consumption units, with producers requiring little or no coproduction from the collective consumption unit.

The framework we consider applies most readily to environments which have separate producers and collective consumption units, thus we focus on environments described in (2) and (3) above.

Mechanisms such as the one we propose may not be necessary in the context of intra-household decision-making (and other environments described by (1)) because of the altruistic preferences that decision-makers have for other members of their household [1]. Our analysis assumes that all decision-makers are completely self-interested but able to cooperate in the crafting and signing of binding agreements. If, alternatively, a group of decision-makers had altruistic preferences for the others' consumptions, mechanisms like the ones we propose may not be necessary to solve collective action problems.[3]

As a simple example, consider a husband and wife who face a collective action problem. Suppose that utilities are cardinal and interpersonally comparable, and that the husband has an "altruistic" utility function in which his utility is the sum of his own consumption utility and his wife's consumption utility. Similarly, suppose the wife also has an altruistic utility function which sums her own consumption utility and her husband's consumption utility.[4] This collective action problem is trivial, since the husband's and wife's individually-rational consumption plans are also efficient from society's perspective; they couldn't be improved upon by a mechanism such as the one we propose below.[5]

We conclude this brief discussion of Ostrom's theoretical insights with a quote that neatly summarizes the need for analytical complements to the received (non-cooperative) theory of collective action:

> "[Both the experimental evidence and] the evidence from field settings show that individuals temporarily caught in a social-dilemma structure

[3]Mechanisms such as the one we propose may even be harmful in such environments. See the discussion in Section 5.

[4]This example is inspired by a similar example in Pollak [25].

[5]Furthemore, the VCG mechanism, on which our analysis relies, is not incentive compatible when the consumers and producers are the same group. See Section 3.

are likely to invest resources to innovate and change the structure itself in order to improve joint outcomes. They also strengthen the earlier evidence that the currently accepted, non-cooperative game theoretical explanation relying on a particular model of the individual does not adequately predict behavior in one-shot and finitely repeated social dilemmas. Cooperative game theory does not provide a better explanation. Since both cooperative and non-cooperative game theory predict extreme values, neither provides explanations for the conditions that tend to enhance or detract from cooperation levels." [22, p. 9]

3 How To Study a Public Economy (II)

In this section, we study a public economy in a way that builds on the insights by Ostrom and her coauthors as spelled out in the previous section. The purpose of this section is to argue that *a mixture* of non-cooperative game theory and cooperative game theory provides an adequate language in which to study two of the main open questions in this literature: (*i*) why some societies are able to solve their collective action problems and others are not, and (*ii*) why societies choose the particular institutions they choose from a vast array of possible choices.

We wish to show how to build models to study public economies by elaborating on the insights by Ostrom and her coauthors in the context of a specific situation. In this paper, we focus on an environment in which there are two competing providers of local public goods of different qualities, and there is heterogeneity of abilities of the potential users to profit from those public goods. Very loosely speaking, these users can be thought of as households, and these providers can be thought of as "clans, neighborhood associations, communal organizations, trade associations, buyers and producer's cooperatives, local voluntary associations and clubs, special districts, international regimes, public service [organizations], arbitration and mediation associations, and charitable organizations." [23, p. 36]

Assume that the *status quo* situation is one of underprovision of the public goods, and that the set of possible reforms includes one that guarantees a (first-best) provision. The questions we ask are: (*i*) What determines whether this society is able to avoid the situation of critical underprovision? More concretely, is it more or less likely for the situation of underprovision to be avoided as the mean ability of the population increases? (*ii*) If the situation of underprovision cannot be avoided, what determines which particular institution is chosen? (*iii*) Is it at least an 'improvement' over the *status quo*?

3.1 The Setup

We consider an environment in which there are two local public (capital) goods, 1 and 2, and a large quantity I of households that have to choose which (if either) of these two goods to use in their household production. Household $i's$ payoff from using good j (which is denoted in the same units as the prices of the public goods) is given by

$$\pi(i,j) = K_j \cdot \theta_i \cdot L_i - p_j,$$

where K_j denotes the intrinsic quality of good j, θ_i denotes household $i's$ productivity (drawn from the uniform distribution on $\left[\underline{\theta},\overline{\theta}\right]$, with $\underline{\theta}> 0$), L_i denotes the household's labor input to production and p_j denotes the price (if any) that households have to pay to have access to good j. Households also have a choice of an outside option ("resource 0") valued at $K_0 \cdot \theta_i \cdot L_i$ by individual i. For simplicity, we assume that households supply one unit of labor inelastically ($L_i = 1, \forall i$). Furthermore, we make the following assumptions about the structure of household profits and on the distribution of productivities:

(A1) $K_2 > K_1 > K_0 > 0$

(A2) $\frac{2+\overline{\theta}-\underline{\theta}}{2\left(\overline{\theta}-\underline{\theta}\right)} < (K_2 + K_1 - 2K_0) < 1$

(A3) $\max\left\{\frac{1}{\overline{\theta}-\underline{\theta}}, \frac{1}{2\underline{\theta}-\overline{\theta}}\right\} < (K_2 - K_1)$

(A4) $\frac{3\left(\overline{\theta}-\underline{\theta}\right)(K_2-K_1)+2}{\left(2\overline{\theta}-\underline{\theta}\right)(K_2-K_1)+1}\frac{\underline{\theta}(K_2-K_1)}{\left(\overline{\theta}-\underline{\theta}\right)(K_2-K_1)+1} < \frac{(K_2-K_1)}{(K_1-K_0)} < \frac{3\left(\overline{\theta}-\underline{\theta}\right)(K_2-K_1)+2}{\left(\overline{\theta}-2\underline{\theta}\right)(K_2-K_1)+1}\frac{\underline{\theta}(K_2-K_1)}{\left(\overline{\theta}-\underline{\theta}\right)(K_2-K_1)+1}$

Assumption $A1$ reveals the ordering of the quality of the goods. Assumption $A2$ imposes bounds to the rewards from switching to a higher quality good. Assumption $A3$ is a lower bound on the rewards from switching to good 2 from good 1. Assumption $A4$ says that the rewards from switching to good 2 from good 1 are not too distant to the rewards from switching to good 1 from the outside option. For these assumptions to hold simultaneously, it is necessary that $\left(\overline{\theta}-\underline{\theta}\right) > 3$ as well, and we assume this, too. Let $v_j(\theta)$ be the reservation price for good j of a household with productivity θ. It is a routine matter to show that

$$\begin{aligned} v_1(\theta) &= \theta(K_1 - K_0) \text{ and} \\ v_2(\theta) &= \theta(K_2 - K_0). \end{aligned}$$

Reservation prices are instrumental in determining the quantity of households that will choose either local public good or the outside option. Let $\mu_j = \mu_j(p_1, p_2)$ be the proportion of households that choose good j at prices p_1, p_2 and $j = 0, 1, 2$.

Associated with each local public good j, there is a producer j, whose profits we denote ρ_j, who may or may not be able to charge a price for the good she produces, yet she bears the costs of producing the good. We assume that the cost of producing good j when used by a proportion μ_j of I households is given by $c_j(\mu_j) = \frac{I}{2}\mu_j^2$. A critical assumption that we will maintain throughout this analysis is that producers can only receive revenues in the form the payments from households using her good. In particular, producers cannot receive payments from households not using her good, or from the other producer.

3.2 The Status Quo Case: Non-Excludability

Legal or technological considerations may dictate that no household can be excluded from the consumption of their good of choice. We model this situation as one in which the prices that the producers can effectively impose on the users of the goods are equal and such that profits vanish.

In this case, for each household i of type θ in $[\underline{\theta}, \overline{\theta}]$, $\pi(i,2) = \pi_2 \cdot \theta_i > \pi(i,1) = \pi_1 \cdot \theta_i > \pi(i,0) = \pi_0 \cdot \theta_i$. As a consequence, all households choose good 2 ($\mu_2^s = 1, \mu_1^s = \mu_0^s = 0$) and, by definition, producer 2 obtains profits equal to 0 (therefore, $\rho_2 = \rho_1 = 0$). This situation with "too many" households using resource 2 and "too few" households using resource 1, is the one that we associate with the well-known "tragedy" results from the literature on public goods and common-pool resources. To make this suboptimality argument precise, we now turn to an examination of the first-best allocation in this situation.

3.3 The "First-Best"

As usual, an allocation μ^f of households to either local public good or to their outside options is a (first-best) efficient allocation whenever μ^f maximizes the sum of the payoffs of the households (including the producers) and subject to the relevant technological constraints. This amounts to the choice of a function $\mu^f(i) : I \to \{0, 1, 2\}$ that assigns every household i in I to either good 1, 2, or to the outside option 0. It is not hard to see that, because of the special structure of this problem, such a function can be written as

$$\mu^f(i) = \begin{cases} 2 & \text{if} \quad \theta_i \in \left[\theta_2^f, \overline{\theta}\right] \\ 1 & \text{if} \quad \theta_i \in \left[\theta_1^f, \theta_2^f\right] \\ 0 & \quad\text{otherwise,} \end{cases}$$

where θ_1^f and θ_2^f solve the problem below.

$$\max_{\theta_1, \theta_2} K_0 \frac{\theta_1 - \underline{\theta}}{\overline{\theta} - \underline{\theta}} I + K_1 \frac{\theta_2 - \theta_1}{\overline{\theta} - \underline{\theta}} I + K_2 \frac{\overline{\theta} - \theta_2}{\overline{\theta} - \underline{\theta}} I - \frac{I}{2} \left(\frac{\theta_2 - \theta_1}{\overline{\theta} - \underline{\theta}} \right)^2 - \frac{I}{2} \left(\frac{\overline{\theta} - \theta_2}{\overline{\theta} - \underline{\theta}} \right)^2$$

Given function $\mu^f(i)$ one can compute the proportion μ_j^f of households that ought to be assigned to good j according to the first-best principle. From the first-order (necessary and sufficient) conditions for the solution of this problem one obtains

$$\begin{aligned} \mu_1^f &= K_1 - K_0 \\ \mu_2^f &= K_2 - K_0, \text{ and} \\ \mu_0^f &= 1 - (K_2 + K_1 - 2K_0), \end{aligned}$$

where all values are strictly between 0 and 1 because of assumptions $A1$ and $A2$. These proportions reveal that the *status quo* situation, that of non-excludability, is indeed one of underprovision of local public good 1.

It is convenient to define

$$FB(\theta) = \sum_{i \in I} K_{\mu^f(i)} \cdot \theta_i - \frac{I}{2} \left(\mu_1^f\right)^2 - \frac{I}{2} \left(\mu_2^f\right)^2$$

as the maximized value of this first-best problem given the profile θ of productivities and $FB_{-i}(\theta)$ as the maximized value of the problem for households other than i from an efficient allocation when the profile of abilities is given by θ.

3.4 Envisioning Reform

The producers of the local public goods are not doing so well in the *status quo* regime: producers 1 and 2 are receiving zero profits. Rather than playing the *status quo* game, the producers face the challenge of collectively choosing an alternative game in which they could fare better. To illustrate how this process of collective choice may be studied, we endow the producers (and *only* the producers) with opportunities for choosing which game to play out of a small set that contains the *status quo* game. Before we continue with the explanation of how the process of game selection takes place, we introduce the set of games from which the players will collectively choose.

3.5 The First Reform: Excludability Through Prices

In the institutional setup corresponding to the first reform, both producers are able to transform the legal and technological barriers that prevented them from effectively discriminating among types of users in the *status quo* game. The new rules are such that each producer chooses prices *independently*, however. As this is very important for the analysis, we stress that it is *not* feasible for producers to choose prices jointly. As before, it is also *not* feasible for the producers to transfer profits among themselves.

An analysis of a situation formally similar to the one depicted here was performed by Gabszewicz and Thisse [7] for the case of zero costs in the context of oligopolistic competition. Their method of analysis can be used to compute the proportion of households that will use either local public good and the outside option given positive prices. In what follows, we will be interested in prices that support a positive proportion of households selecting both goods, and no proportion of households selecting the outside option. Let θ^e be such that

$$v_2(\theta^e) - p_2 = v_1(\theta^e) - p_1,$$

and consider the conjecture, subject to verification, that

$$p_2 > v_1\left(\underline{\theta}\right) > p_1$$

By construction, given prices p_1 and p_2, all households with productivities in the interval $[\underline{\theta}, \theta^e]$ will choose good 1 and all households with productivities in the interval $\left[\theta^e, \overline{\theta}\right]$ will choose good 2. Therefore, the proportions of households that will use each good in this scenario as a function of prices are given by

$$\begin{aligned}
\mu_1\left(p_1, p_2\right) &= \frac{p_2 - p_1}{\left(K_2 - K_1\right)\left(\overline{\theta} - \underline{\theta}\right)} - \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}}, \\
\mu_2\left(p_1, p_2\right) &= 1 - \frac{p_2 - p_1}{\left(K_2 - K_1\right)\left(\overline{\theta} - \underline{\theta}\right)} + \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}} \text{ and} \\
\mu_0\left(p_1, p_2\right) &= 0.
\end{aligned}$$

The proof of this fact is virtually identical to the proof of Lemma 2 in Gabszewicz and Thisse [7] and we therefore omit it here.

With this information about the proportion of households that will select good 1, the problem of each producer j is to select the price p_j^e that maximizes her profits, when taking the price p_{-j}^e chosen by the other producer as given. Therefore, p_1^e and

p_2^e are a solution to

$$\max_{p_1} \; p_1 \cdot \mu_1^e\left(p_1, p_2^e\right) I - \frac{I}{2}\mu_1^e\left(p_1, p_2^e\right)^2 \text{ and}$$
$$\max_{p_2} \; p_2 \cdot \mu_2^e\left(p_1^e, p_2\right) I - \frac{I}{2}\mu_1^e\left(p_1^e, p_2\right)^2 \; .$$

From the strict concavity of the objective functions of the producers, it follows that their best-response functions are unique and given by

$$\begin{aligned} p_1 &= \frac{1-a}{b}\frac{1+b}{2+b} + \frac{1+b}{2+b}p_2 \text{ and} \\ p_2 &= \frac{a}{b}\frac{1+b}{2+b} + \frac{1+b}{2+b}p_1, \end{aligned}$$

where $b = \frac{1}{\left(\overline{\theta}-\underline{\theta}\right)(K_2-K_1)} < 1$ and $1 < a = \frac{\overline{\theta}}{\overline{\theta}-\underline{\theta}} < (2+b)$ (by assumptions $A1 - A2$). This pair of equations for the prices have a unique solution p_1^e and p_2^e given by

$$\begin{aligned} p_1^e &= \frac{1+b}{b}\frac{2+b-a}{3+2b} \text{ and} \\ p_2^e &= \frac{1+b}{b}\frac{a+1+b}{3+2b}. \end{aligned}$$

Notice that both prices are strictly positive and that $p_2^e > v_1\left(\underline{\theta}\right) > p_1^e$, as conjectured. A proof of this can be found in the Appendix.

With these results in hand, one can compute the proportion of households that will select each good in equilibrium

$$\begin{aligned} \mu_1^e &= \frac{2+b-a}{3+2b}, \\ \mu_2^e &= \frac{a+1+b}{3+2b} \text{ and} \\ \mu_0^e &= 0. \end{aligned}$$

and the profits that both producers collect

$$\begin{aligned} \rho_1^e &= \frac{2+b}{2b}\left(\frac{2+b-a}{3+2b}\right)^2 I \text{ and} \\ \rho_2^e &= \frac{2+b}{2b}\left(\frac{a+1+b}{3+2b}\right)^2 I. \end{aligned}$$

These profits, therefore, are what the producers can expect to obtain if this is the game that they choose to play.

3.6 The Second Reform: Implementing the First-Best Outcome

The second reform that we will allow in the set of possible reforms is an institutional setup that *requires* the producers to implement the efficient (first-best) outcome described earlier. This implementation is naturally not a trivial matter because the producers of the goods neither know the productivities of the households in the economy, nor can they force the households to participate in the institution they design to elicit an efficient consumption of the goods. As a consequence, inducing them to reveal the information necessary for the implementation of the efficient outcome through appropriate compensation can be so costly to the producers that it may make it undesirable, from their point of view, to implement it at all. The point, therefore, of allowing the producers the option to bond themselves to the efficient production of the public goods is to see whether they would, jointly and voluntarily, take this option in a situation of cooperative bargaining.

The setup is, to be sure, one in which each producer is required to select a method for inducing a (first-best) efficient consumption of the public good under her administration. The method may or may not involve communication of any kind between any of the participants in the economy and may or may not involve compensation contingent on the content of such communication. In this scenario for reform, each producer has the ability to arbitrarily exclude any household from employing the public good under her administration, yet each producer is not allowed to accept compensation either from the other producer nor from households using the good that is not under her administration.

It turns out that there is an easy way to characterize what would happen in such a situation. *It is a dominant strategy for each producer to use the Vickrey-Clarke-Groves (VCG) mechanism (among the class of efficient mechanisms).* This is due to a result by Krishna and Perry [16], who show that this mechanism maximizes the payments collected from *each* household participating in the mechanism among the class of (first-best) efficient mechanisms.

We review the construction of this mechanism briefly following the approach in Krishna and Perry [16]. The *VCG mechanism with basis* $\underline{\theta}$, denoted by $\left(\mu^f, p^v\right)$, is defined as one in which each household i is invited to give a report r_i of its productivity, is assigned to a public good, and is charged a price for the employment of this public good that is contingent on the report. The payments required from each household willing to participate in the mechanism are given by

$$p^v(\mathbf{r},i) = [FB(\underline{\theta}, \mathbf{r}_{-i}) - FB_{-i}(\mathbf{r})] - K_0 \cdot \underline{\theta},$$

where $FB(\underline{\theta}, \mathbf{r}_{-i})$ is the (maximized) value of the first-best problem given the profile

$(\underline{\theta}, \mathbf{r}_{-i})$ of productivities and $FB_{-i}(\mathbf{r})$ as the (maximized) value of households other than i from an efficient allocation when the profile of productivities is given by $\mathbf{r}$. Notice that the first best rule μ^f assigns a household with ability $\underline{\theta}$ to its outside option. Therefore, a household i with productivity $\underline{\theta}$ is the "most reluctant" type of agent i in the sense that its gain from participating in the VGG mechanism is the least among all the types of i. As a consequence, the payment rule for household i can be written as

$$p^v(\mathbf{r},i) = [K_0 \cdot \underline{\theta} + FB_{-i}(\underline{\theta}, \mathbf{r}_{-i}) - FB_{-i}(\mathbf{r})] - K_0 \cdot \underline{\theta} = FB_{-i}(\underline{\theta}, \mathbf{r}_{-i}) - FB_{-i}(\mathbf{r}).$$

The amount $p^v(\mathbf{r},i)$ represents the *externality* that i exerts on the rest of the economy by being of productivity r_i rather than $\underline{\theta}$. It is the difference between the welfare of others "without household i" and the welfare of others "with household i." For the problem at hand the payment rule is

$$p^v(\mathbf{r},i) = \begin{cases} p_2^v = \theta_1^f (K_1 - K_0) + \theta_2^f (K_2 - K_1) & \text{if } \; r_i \in \left[\theta_2^f, \overline{\theta}\right] \\ p_1^v = \theta_1^f (K_1 - K_0) & \text{if } \; r_i \in \left[\theta_1^f, \theta_2^f\right] \\ p_0^v = 0 & \text{otherwise.} \end{cases}$$

It is not hard to see that truth-telling is a weakly dominant strategy in this mechanism and thus it is also incentive compatible. This is proved in the Appendix. The profits that each producer gets from using a pricing scheme as in this mechanism are given by

$$\begin{aligned} \rho_1^v &= p_1^v \mu_1^f I - \frac{I}{2}\left(\mu_1^f\right)^2 \text{ and} \\ \rho_2^v &= p_2^v \mu_2^f I - \frac{I}{2}\left(\mu_2^f\right)^2, \end{aligned}$$

or

$$\begin{aligned} \rho_1^v &= \left(\theta_1^f - \frac{1}{2}\right)(K_1 - K_0)^2 I \text{ and} \\ \rho_2^v &= \left[\left(\theta_1^f - \frac{1}{2}\right) + \left(\overline{\theta} - \underline{\theta}\right)(K_2 - K_1)\right](K_2 - K_0)^2 I. \end{aligned}$$

3.7 The Grand Game: A Mixture of Cooperative and Non-Cooperative Behavior

Thus far, we have described the *status quo* situation, "non-excludability,"as well as two potential reforms, described above, "excludability" and "first-best reform." We propose a two-stage supergame. In the first stage, the players cooperatively choose, according to Nash's rules of axiomatic bargaining,[6] which game to play, while in the second stage they play a Nash equilibrium of the chosen game. We call this *the Nash-Nash solution of the public economy problem.* Formally, G^s is the *status quo* game described above, and G^e and G^v stand for the excludability case and the first-best reform, respectively. The Nash-Nash solution of the public economy problem under study then requires the application of the Nash bargaining solution over the Nash equilibrium payoffs of the games G^s, G^e and G^v, with the disagreement payoffs given by the equilibrium payoffs of the *status quo* game, G^s.

3.7.1 Analysis

As there is not a straightforward manner of comparing the products $\rho_1^v(\rho_2^v - \rho_2^s)$ and $\rho_1^e(\rho_2^e - \rho_2^s)$ for computing this solution in general, we consider evaluating those products along the curves $(\rho_1^v(m), \rho_2^v(m))$ and $(\rho_1^e(m), \rho_2^e(m))$ as the mean m of distribution of productivities (keeping the variance constant) varies in the interval $[\underline{m}, \overline{m}]$, where $\underline{m}$ and $\overline{m}$ must satisfy $\left(\frac{3}{2}+b\right)\left(\overline{\theta}-\underline{\theta}\right) \geq \overline{m} > \underline{m} \geq \frac{3}{2}$ because of $A1-A4$. It turns out that, for m close enough to $\underline{m}$, $\rho_1^v(m)$ is negative, so that producer 1 will always reject any proposal to select game G^v because she does better in the status quo game G^s, where she obtains zero profits. Interestingly, the product $\rho_1^v(\rho_2^v - \rho_2^s)$ is strictly increasing in m and the product $\rho_1^e(\rho_2^e - \rho_2^s)$ is strictly decreasing in m. Indeed, as m approaches $\overline{m}$, profits $\rho_1^e(m)$ approach zero, and therefore the product $\rho_1^e(\rho_2^e - \rho_2^s)$ approaches zero as well. Thus, there is a mean income m^* such that if $m > m^*$ the Nash-Nash solution of the public economy problem picks game G^v and if $m < m^*$ the Nash-Nash solution picks game G^e. Details are given in Appendix 2.

4 Discussion

With this analysis in mind, we are ready to answer the specific theoretical questions posed at the beginning of this section, as inspired from the field work by Ostrom and coauthors: (i) What determines whether this society is able to avoid the situation of critical underprovision? More concretely, is it more or less likely for the situation of underprovision to be avoided as the mean ability of the population increases? (ii) If the situation of underprovision cannot be avoided, what determines which particular institution is chosen? (iii) Is the outcome of the process of institution selection at least an 'improvement' over the *status quo*?

[6] A justification for the application of Nash's bargaining solution in this setting can be found in Kaneko [13].

(i) The situation of critical underprovision can arise if the revenue raised by producer 1 in game G^v is so small that it would force producer 1 to run a deficit in order to induce an efficient consumption of good 1. This will happen if the mean productivity of the potential users of the goods is low. Therefore, the situation of underprovision of the best good is more likely to be avoided as the mean productivity of the population of households increases.

In the context of the model, question (ii) becomes: Which non-cooperative game will be selected cooperatively as the Nash-Nash solution of the public economy problem? Interestingly, the first-best efficient outcome need not be selected as soon as producer 1 ceases to run a deficit. This is because the profits that she obtains in the excludability case may still be much higher than those she obtains in the efficient allocation game. Interestingly, as the mean productivity increases, there are two effects that weaken the appeal of the excludability case in favor of the first-best case.

The first reason that the first-best case becomes more attractive as mean productivity increases is that the profits of both producers are increasing in mean productivity in the first-best situation. This is so because the first-best allocation, and therefore costs, do not depend on the mean productivity, but willingness to pay for the consumption of the goods increases uniformly across households with the mean productivity. This simply means that more revenues are available for the producers with no added costs. The second reason is that the profits of producer 2 are increasing in mean productivity but the profits of producer 1 actually *decrease* with mean productivity in the excludability case. This is so because, with an increase in mean productivity, every household's valuation of both public goods increases, but the valuation of the best good increases relatively more than that of the worst good. This increase in relative valuation leads to an increase in *both* the price charged by producer 2 and the proportion of households using good 2. The combination of these two effects jointly explains how changes in m affect which non-cooperative game will be selected cooperatively as the Nash-Nash solution of the public economy problem.

Finally, we address question (iii) : Is the outcome of the collective choice process an improvement over the status quo? Not surprisingly, *it depends on who counts* in the computation of welfare, relative to who is a participant in the collective choice process of institutional design. The excludability case does not yield a first-best allocation of households to all goods, but it yields a smaller underprovision of the best good and therefore an increase in efficiency. In spite of this, however, everybody in this economy, *except for the producers*, is worse off under either reform when we compare their payoffs to those which obtain in the *status quo* situation. This is a reminder of the fact that, when the opportunities for redistribution are severely restricted and not all households have equal access to participation in the collective choice process, there is no reason to believe that an increase in efficiency will be directly linked to the overall welfare of society, a point that has been made in other contexts, such as by Hammond [10], Ray and Vohra [26] and Moulin [18].

4.1 The Role of Altruism

Our model assumes that households behave without regard for their peers. This assumption is, of course, unrealistic, as other-regarding preferences such as altruism are both wide-spread [11] and economically significant (e.g. [5]). However, the finding that decision makers are not, on average, completely selfish does not necessarily indicate that institutions should be designed with other-regarding preferences in mind. Brennan and Buchanan [4], for example, believe that a realistic model of (average) human behavior should *not* factor into normative institutional design. Institutions, they believe, should instead be designed with those that are most likely to abuse the institutions in mind. This belief was originally espoused by moral philosophers like David Hume [12] and John Stuart Mill [19].[7] These authors believed that institutions should be devised so as to minimize the amount of damage that particularly self-interested members of the collective consumption unit could inflict.

A number of authors have called into question this approach to institutional design. Frey pushes back on Brennan and Buchanan (in the context of designing a constitution) by stating, "a constitution designed for knaves tends to drive out civic virtues. As a result, the constitution is less observed. The effort to guard the constitution against exploitation may thus lead to a perverse result" [6, p. 44]. In contrast to Brennan and Bucanan, Frey believes that if a society wants its members to act altruistically, it should design institutions that allow them to behave as such. Recent empirical studies in behavioral and experimental economics support this position. These studies show that introducing formal mechanisms (such as ours) in environments that previously relied on informal mechanisms based on altruism, can have the opposite effect as the prediction of neoclassical economics. Bowles reviews 41 experiments and ultimately finds ample "experimental evidence that some mechanisms induce even the civic-minded to act as if they were selfish" [3, p. 1609].[8]

[7] Hume, for example, states that "[i]n contriving any system of government, and fixing the several checks and controls of the constitutions, every man ought to be supposed a knave, and to have no other end, in all his actions, than private interest" [12, p. 117-18].

[8] Vatn [27] has also explored the link between institutions and altruism, though his approach is different than that of Frey and Bowles. Vatn emphasizes that social interactions are complex, and he believes that institutions help alleviate this complexity by showing decision makers how they *should* behave. Institutions "may define which kind of reasoning is expected or more specifically which kind of acts is required" [27, p. 12]. In some cases, an institution may indicate that decision

In summarizing the findings of this literature, Gneezy et al. explain that "[w]hen explicit incentives seek to change behavior in areas like education, contributions to public goods, and forming habits, a potential conflict arises between the direct extrinsic effect of the incentives and how these incentives can crowd out intrinsic motivations in the [short run] and the long run" [8, p. 206]. For example, in a canonical experiment by Gneezy and Rustichini [9], a group of daycare providers began charging parents a small fee of 10 Israeli new shekels (about $3 at the time) for picking-up their children late (as opposed to an old system in which no late fee was charged). The researchers found that, contrary to the predictions of the standard model of neoclassical economics, this fee increased the proportion of late pick-ups rather than decreasing it. One interpretation of this result relies on social norms. Before the fee was implemented, parents picked-up their children on time in order to avoid the guilt associated with taking advantage of the daycare workers. However, after the fee was implemented, any previous feelings of guilt were alleviated by paying a fee to compensate for a late pick-up. Parents who would pay more than 10 shekels to avoid this guilt would be more likely to pick-up their children late after the fee was implemented.

We wish to make one final point in discussing the role of incentives and altruism in the context of institutional economics, as this literature provides clues as to the types of institutions that our framework is likely to describe. Environments in which it only takes one "knave" to cause tremendous damage are more likely to be designed with knaves in mind, in the tradition of Brennan and Buchanan [4], Hume [12], and Mill [19]. Likewise, institutions in which the scope of the damage that one knave can do are limited, but the potential gains from altruistic behavior are significant, are likely to be designed to take advantage of altruism, as advocated by Frey [6], Gneezy et al. [8], and Bowles [3]. Our framework explicitly assumes that all decision makers are purely self-interested—that is, our model economy is populated with knaves—and, as such, our mechanism seems most likely to be successful in the types of environments discussed by Brennan and Buchanan and their luminaries.

At the level of abstraction we consider, there is no restriction on the size of the collective consumption unit to which our model could apply. However, as discussed in sub-section 2.2, very small (and very close-knit) collective consumption units such as individual households are likely to solve collective action problems through altruism rather than through mechanisms like ours. However, the reliance on altruism is likely to dwindle as collective consumption units grow large. We believe our model applies most readily to collective consumption units that are large enough that they

makers should be reasoning as if they are maximizing own utilities, while other institutions may indicate decision makers should be maximizing the group's payoffs.

cannot realistically be represented with models of altruism, but small enough that cooperation at the "rules-setting" stage of the game is feasible.

5 Conclusions

The point of our exercise is to illustrate how to take a public economy, as defined and studied in the field by Ostrom and her coauthors, and study from a theoretical standpoint the institutional outcomes one may expect to see from an examination of how those economies engage in their collective choice of institutions. In line with Ostrom's insights, this approach arises from the blending of cooperative and non-cooperative approaches to game theory in designing a formalism in which first the players with political agency cooperate to design the rules of the game they wish to play, and then, at a later stage, make independent, non-cooperative choices at the moment of playing the (collectively) chosen game. Once the formalism is in place, one can answer questions with it such as the ones that motivate those who study the common resource management community, such as: (*i*) Will reform take place? (*ii*) Will reform be an improvement over the status quo? and (*iii*) How are the gains and losses from reform distributed among the members of society?

To illustrate this approach, we applied the modeling strategy in this paper to a situation in which two providers compete in the provision of local public goods of different quality and there is heterogeneity of abilities of the potential users to profit from those public goods. A careful examination of the incentives that the users and producers of the local public goods have in the equilibrium of the public economy allows us to obtain specific answers to those questions, in the context of the particular example at hand.

While the specific results that our analysis generates are interesting in their own right, as described in detail in sub-section 4, what we wish to stress is the general methodology we employ, namely, that a blend of the cooperative and non-cooperative approaches to game theory can be used to formalize the insights developed by Ostrom and her coauthors in her many field and experimental studies. We believe that this methodology can be used to provide further economic insight into the problems that are still open in the critically important field of common pool resource management.

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6 Appendix

6.1 Proof that $0 < p_1^e < v_1(\underline{\theta}) < p_2^e$ in the first reform

The price p_j^e that producer j charges to users of good j in a non-cooperative price competition scenario is given, in equilibrium, by

$$\begin{aligned} p_1^e &= \frac{1+b}{b}\frac{2+b-a}{3+2b} \text{ and} \\ p_2^e &= \frac{1+b}{b}\frac{a+1+b}{3+2b}. \end{aligned}$$

Clearly, $p_2^e > 0$. Price p_1^e is also positive because $A1 - A3$ guarantee that $a < 2 + b$.

We now verify that $p_1^e < v_1(\underline{\theta}) < p_2^e$. From assumption $A4$ we have that

$$\frac{3+2b}{a+1+b}\frac{a-1}{1+b} < \frac{(K_2 - K_1)}{(K_1 - K_0)} < \frac{3+2b}{2+b-a}\frac{a-1}{1+b}.$$

Rearranging this expression one obtains

$$\frac{3+2b}{a+1+b}\frac{b}{1+b} < \frac{1}{(K_1 - K_0)\,\underline{\theta}} < \frac{3+2b}{2+b-a}\frac{b}{1+b},$$

which can be written as

$$\frac{(1+b)}{b}\left(\frac{2+b-a}{3+2b}\right) = p_1^e < \underline{\theta}\,(K_1 - K_0) = v_1(\underline{\theta}) < \frac{(1+b)}{b}\left(\frac{a+1+b}{3+2b}\right) = p_2^e,$$

This is what we wanted to show.

6.2 Proof that participating and truth-telling is a weakly dominant strategy for the households in the second reform

For the problem at hand the payment rule is

$$p_i^v(\mathbf{r}) = \begin{cases} \theta_1^f (K_1 - K_0) + \theta_2^f (K_2 - K_1) & \text{if } r_i \in \left[\theta_2^f, \overline{\theta}\right] \\ \theta_1^f (K_1 - K_0) & \text{if } r_i \in \left[\theta_1^f, \theta_2^f\right] \\ 0 & \text{otherwise,} \end{cases}$$

Consider a household i with productivity $\theta_i \in \left[\underline{\theta}, \theta_1^f\right]$. If the household reports its true productivity it gets to pay nothing, is assigned to good 0 and ends up with a payoff equal to $K_0 \cdot \theta_i > 0$. If the household reports any other productivity in the interval $\left[\underline{\theta}, \theta_1^f\right]$ it gets exactly the same payoff as before, so it has no incentives to deviate to any such report. If the household reports a productivity in the interval $\left(\theta_1^f, \theta_2^f\right]$ it is assigned to good 1, pays a price equal to $\theta_1^f (K_1 - K_0)$ and gets a payoff equal to $K_1 \cdot \theta_i - \theta_1^f (K_1 - K_0) = K_1 \left(\theta_i - \theta_1^f\right) + \theta_1^f (K_0)$. This payoff is smaller than that obtained by reporting its true type, however, since $K_1 \left(\theta_i - \theta_1^f\right) + \theta_1^f \cdot K_0 - K_0 \cdot \theta_i = (K_1 - K_0) \left(\theta_i - \theta_1^f\right) < 0$. If the household reports a productivity in the interval $\left(\theta_2^f, \overline{\theta}\right]$ it is assigned to good 2, pays a price equal to $\theta_1^f (K_1 - K_0) + \theta_2^f (K_2 - K_1)$ and gets a payoff equal to $K_2 \cdot \theta_i - \theta_1^f (K_1 - K_0) - \theta_2^f (K_2 - K_1)$. The difference between this payoff and the payoff that obtains from reporting a productivity in the interval $\left(\theta_1^f, \theta_2^f\right]$ is given by

$$K_1 \cdot \theta_i - \theta_1^f (K_1 - K_0) - \theta_2^f (K_2 - K_1) - K_1 \left(\theta_i - \theta_1^f\right) - \theta_1^f (K_0),$$

which can be written as $(K_2 - K_1) \left(\theta_i - \theta_2^f\right) < 0$. Therefore, reporting an ability in the interval $\left(\theta_2^f, \overline{\theta}\right]$ is a dominated strategy. From all this, it follows that truth-telling is a weakly dominant strategy. Also, since the household receives a payoff of at least the value of its outside option, the participation constraint is also met.

The arguments for households with productivities in the intervals $\left(\theta_1^f, \theta_2^f\right]$ and $\left(\theta_2^f, \overline{\theta}\right]$ to show that truth-telling is a dominant strategy are analogous to the one just given, and we omit them here.

6.3 Details on the effect of changes in mean productivity m on the Nash-Nash solution of the public economy problem

Since the mean productivity m is defined as $\left(\frac{\overline{\theta}+\underline{\theta}}{2}\right)$, the terms $\overline{\theta}$ and $\underline{\theta}$ can be defined as $\overline{\theta} = m + \frac{1}{2}\left(\overline{\theta} - \underline{\theta}\right)$, $a = \frac{m}{\left(\overline{\theta}-\underline{\theta}\right)} + \frac{1}{2}$ and $\theta_1^f = m + \left(\overline{\theta} - \underline{\theta}\right)\left[\frac{1}{2} - (K_1 + K_2 - 2K_0)\right]$for fixed $\left(\overline{\theta} - \underline{\theta}\right)$. In what follows all functions of m are assumed to have domain equal to $[\underline{m}, \overline{m}]$. We can therefore compute the expressions

$$\begin{aligned}
\frac{\partial \rho_1^e}{\partial m} &= -\frac{2+b}{b}\left(\frac{2+b-a}{3+2b}\right) I \left(\frac{1}{\left(\overline{\theta}-\underline{\theta}\right)(3+2b)}\right) < 0, \\
\frac{\partial \rho_2^e}{\partial m} &= \frac{2+b}{b}\left(\frac{a+1+b}{3+2b}\right) I \left(\frac{1}{\left(\overline{\theta}-\underline{\theta}\right)(3+2b)}\right) > 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \rho_1^v}{\partial m} &= (K_1 - K_0)^2 I > 0 \\
\frac{\partial \rho_2^v}{\partial m} &= (K_2 - K_0)^2 I > 0.
\end{aligned}$$

Notice that the product $\rho_1^e \rho_2^e$ is continuous and strictly decreasing in m

$$\begin{aligned}
\frac{\partial \rho_1^e \rho_2^e}{\partial m} &= \frac{\partial \rho_1^e}{\partial m}\rho_2^e + \frac{\partial \rho_2^e}{\partial m}\rho_1^e \\
&= -\left\{\frac{2+b}{b}\left(\frac{2+b-a}{3+2b}\right) I \left(\frac{1}{\left(\overline{\theta}-\underline{\theta}\right)(3+2b)}\right)\left[\frac{2+b}{2b}\left(\frac{a+1+b}{3+2b}\right)^2 I\right]\right\} \\
&\quad + \left\{\frac{2+b}{b}\left(\frac{a+1+b}{3+2b}\right) I \left(\frac{1}{\left(\overline{\theta}-\underline{\theta}\right)(3+2b)}\right)\frac{2+b}{2b}\left(\frac{2+b-a}{3+2b}\right)^2 I\right\} \\
&= \underbrace{\left[\frac{(2+b)^2}{2b^2}\frac{(a+1+b)(2+b-a)}{(3+2b)^4}\frac{I^2}{\left(\overline{\theta}-\underline{\theta}\right)}\right]}_{[+]}\underbrace{[1-2a]}_{[-]} + \underbrace{\frac{\partial \rho_1^e}{\partial m}}_{[-]} < 0
\end{aligned}$$

and that the product $\rho_1^v (\rho_2^v - \rho_2^s)$ is continuous and strictly increasing in m

$$\frac{\partial \rho_1^v \rho_2^v}{\partial m} = \underbrace{\frac{\partial \rho_1^v}{\partial m}}_{[+]}\rho_2^v + \underbrace{\frac{\partial \rho_2^v}{\partial m}}_{[+]}\rho_1^v > 0$$

Now, evaluate the function $d(m) = \rho_1^e \rho_2^e - \rho_1^v \rho_2^v$. It is continuous and strictly decreasing. Moreover, this function is such that, for m close enough to $\underline{m}$, $d(m) > 0$ and for m close enough to $\overline{m}$, $d(m) < 0$. Then, by the intermediate value theorem, there is a unique m^* such that if $m < m^*$ then $\rho_1^e \rho_2^e > \rho_1^v \rho_2^v$ and if $m > m^*$ then $\rho_1^e \rho_2^e < \rho_1^v \rho_2^v$, that is, the Nash-Nash solution of the public economy problem selects game G^e if $m < m^*$ and game G^v if $m > m^*$.