Economic Theory (2008) 36:147–158
DOI 10.1007/s00199-007-0272-1

RESEARCH ARTICLE

Expected utility inequalities: theory and applications

Eduardo Zambrano

Received: 6 July 2006 / Accepted: 13 July 2007 / Published online: 28 August 2007

Abstract Suppose we know the utility function of a risk averse decision maker who values a risky prospect **X** at a price CE. Based on this information alone I develop upper bounds for the tails of the probabilistic belief about **X** of the decision maker. In the paper I also illustrate how to use these *expected utility bounds* in a variety of applications, which include the estimation of risk measures from observed data, option valuation, and the study of credit risk.

Keywords Expected utility theory · Elicitation of subjective beliefs · Value at risk · Option pricing · Credit risk

JEL Classification Numbers C44 · D81

1 Introduction

Suppose we know the utility function of a risk averse decision maker who values a risky prospect **X** at a price CE. Based on this very limited information, can we know anything whatsoever about the beliefs held by the decision maker about **X**?

I would like to thank John Cochrane, Tom Cosimano, Amanda Friedenberg, George Korniotis, Markus Brunermeier and Paul Schultz for helpful discussions and to participants at two Notre Dame seminars, at the 2006 Spring Midwest Economic Theory and International Economics Conference, and at the 2006 Australasian Meeting of the Econometric Society for their very useful comments. I began working on this project during a year-long visit to the Central Bank of Venezuela. I gratefully acknowledge their hospitality and financial support.

E. Zambrano (✉)
Department of Economics, Orfalea College of Business, Cal Poly,
San Luis Obispo, CA 93407, USA
e-mail: ezambran@calpoly.edu

The surprising answer is that we can, and the reason is that the CE cannot be too high if the decision maker believes with high probability that the risky prospect will perform extremely poorly. Similarly, the CE cannot be too low if the decision maker believes with high probability that the risky prospect will perform extremely well. The main result of this paper, Theorem 1, provides upper bounds for the tails of the probabilistic belief of an expected utility decision maker; bounds that depend on the utility function and on CE, but that do not depend on assumptions made on the shape of the probability distribution that describes the beliefs of the decision maker.

To illustrate the result consider a decision maker with a constant coefficient of absolute risk aversion, given by $1/r$. Assume that this decision maker values a risky prospect $\mathbf{X}$ at a price CE. Is there anything we can say about the beliefs of the decision maker about $\mathbf{X}$ based on this information alone? It turns out that we can: Theorem 1 in this paper implies that this decision maker cannot assign probability greater than 14% to $\mathbf{X}$ taking values less than $2r$ to the left of CE. That is, for this decision maker, $P(\mathbf{X} \leq CE - 2r) \leq 14\%$, an estimate made without making any assumptions about the shape of the distribution of $\mathbf{X}$. The intuition is that the decision maker would have not been willing to pay CE for $\mathbf{X}$ if he or she was too pessimistic about its most likely values. This implies that the certainty equivalent of $\mathbf{X}$ contains information about the subjective probability this decision maker assigns to the extreme events based on $\mathbf{X}$, information that is modulated by the decision maker's attitudes towards risk. I show where the 14% in the example comes from in Sect. 2.3 below.

Upon reflection, the logic behind Theorem 1 is quite clear, and in fact the mathematics behind it are very simple. It is therefore quite surprising that this result had been overlooked, despite the wide scope of applicability that a result of this type may have.[1]

The rest of this paper is devoted to a coherent presentation of the result and its proof, together with introducing applications of these *expected utility bounds* to a variety of economic problems, which include the estimation of risk measures from observed data, option valuation, and the study of credit risk.

2 Theory

2.1 The setup

Consider a risky prospect $\mathbf{X}$ that takes values in a set of possible outcomes $\mathcal{C} \subseteq \mathbb{R}$. The decision maker is risk averse and has preferences over risky prospects given by an increasing and continuous (Bernoulli) utility function $u : \mathcal{C} \rightarrow \mathbb{R}$, known to the analyst, and by some probability distribution function P over the (measurable) subsets of $\mathcal{C}$. This probability distribution is unknown to the analyst.

Suppose that, in addition, the analyst observes that the decision maker's certainty equivalent for $\mathbf{X}$ is equal to CE. The main goal of this paper is to see if we can recover

[1] An exception to this is the work by LiCalzi (2000), who developed upper and lower bounds for the expected utility of a risky prospect based on simple statistics of the risky prospect, like its median or mode. Hence, it can be viewed as complementary to the present work.

some useful information about P from the fact that $\mathbf{X}$ is defined over $\mathcal{C}$, the decision maker has utility function u and values the risky prospect at CE.

Without loss of generality, I normalize the origin and units of u in such a way that $\overline{u} := \sup_{x \in \mathcal{C}} u(x) = 0$, when an upper bound for u exists on $\mathcal{C}$, and $\underline{u} := \inf_{x \in \mathcal{C}} u(x) = -1$, when a lower bound for u exists on $\mathcal{C}$.[2]

Throughout the paper all random variables will be denoted by bold letters.

2.2 The result

The main results of the paper is the following.

Theorem 1 *Suppose* $\mathbf{X}$ *is a risky prospect valued at* CE *by a decision maker with utility function* u. *Then, for* $k > 0$,

(i) *If* $\overline{u} = 0$,

$$P(\mathbf{X} \leq CE - k) \leq \frac{u(CE)}{u(CE-k)}, \quad \textit{and}$$

(ii) *If* $\underline{u} = -1$,

$$P(\mathbf{X} \geq CE + k) \leq \frac{u(CE)+1}{u(CE+k)+1}.$$

Proof Let $s_k = \sup\{u(x) : x \leq CE - k\}$. Since u is increasing, $s_k = u(CE-k)$. The definition of s_k and the fact that $u(x) \leq 0$ for all $x \in \mathcal{C}$ imply that

$$u(CE-k)\, 1_{\{\mathbf{X} \leq CE-k\}} \geq u(\mathbf{X})\, 1_{\{\mathbf{X} \leq CE-k\}} \geq u(\mathbf{X}).$$

Now take expected values to obtain

$$u(CE-k)\ \ P(\mathbf{X} \leq CE-k) \geq Eu(\mathbf{X}).$$

By the definition of the certainty equivalent CE of a risky prospect, $Eu(\mathbf{X}) = u(CE)$. Using this fact and rearranging yields (i).

To prove (ii) let $i_k = \inf\{u(x) : x \geq CE + k\}$. Since u is increasing, $i_k = u(CE+k)$. The definition of i_k and the fact that $u(x) \geq -1$ for all $x \in \mathcal{C}$ imply that

$$u(\mathbf{X}) \geq u(CE+k)\, 1_{\{\mathbf{X} \geq CE+k\}} + (-1)\, 1_{\{\mathbf{X} \leq CE+k\}}.$$

Now take expected values to obtain

[2] To be sure, given any increasing and continuous utility function v with upper and lower bound on $\mathcal{C}$ given respectively by $\overline{v}$ and $\underline{v}$, define u by $u(x) = \frac{v(x)-\overline{v}}{(\overline{v}-\underline{v})}$. It can be checked that $u(x) \leq 0$ for all x and $\underline{u} = -1$ as desired.

$$Eu(\mathbf{X}) = u(CE) \geq u(CE+k)\ \ P(\mathbf{X} \geq CE+k) - [1 - P(\mathbf{X} \geq CE+k)]\,.$$

Rearranging this expression yields (ii). □

Remark 1 An upper bound for u arises naturally, no matter the shape of $\mathcal{C}$, if u is bounded from above, as with constant absolute risk aversion preferences. Alternatively, an upper (resp. lower) bound for u arises, no matter the shape of u, if $\mathcal{C}$ is bounded above (resp. below), as in Rothschild and Stiglitz (1970, 1971).

Remark 2 The reader may notice that the proof of this result works in the same way as that for Chebyshev's inequality, an inequality that provides estimates of the tail of a probability distribution based solely on estimates of its mean and standard deviation. There is a sense in which the theorem above provides an estimate of the lower tail of the distribution in the same way, but by using information about economically meaningful variables, such as CE and u, rather than mainly statistical measures such as the mean and the standard deviation of $\mathbf{X}$.

The example in the next section makes this interpretive point more precise.

2.3 An example with CARA preferences

Consider a decision maker with preferences given by a Bernoulli utility function that satisfies constant absolute risk aversion, that is,

$$u(x) = -e^{-\frac{x}{r}},$$

for $r > 0$. The parameter r in this formulation measures the decision maker's tolerance for risk and it is equal to the inverse of the coefficient of absolute risk aversion for the decision maker.

I now set myself to prove the claim made in the Introduction to this paper: an individual with a constant risk tolerance of r that values a risky prospect $\mathbf{X}$ at a price CE cannot assign probability greater than 14% to $\mathbf{X}$ taking values less than $2r$ to the left of CE. That is, for this decision maker, $P(\mathbf{X} \leq CE - 2r) \leq 14\%$.

To see how this is true simply notice that the utility function being considered is bounded above by zero and hence, by Theorem 1

$$P(\mathbf{X} \leq CE - k) \leq e^{\frac{CE-k-CE}{r}} = e^{-\frac{k}{r}}.$$

Replacing k with zr yields $P(\mathbf{X} \leq CE - zr) \leq e^{-z}$, and setting $z = 2$ gives the desired result, as $e^{-2} \approx 14\%$. This simple fact is summarized in the Lemma below.

Lemma 2 *Suppose* $\mathbf{X}$ *is a risky prospect valued at* CE *by a decision maker with constant risk tolerance parameter given by* r. *Then*

$$P(\mathbf{X} \leq CE - zr) \leq e^{-z} \tag{1}$$

Remark 3 It is instructive to compare this estimate with the one given by the one-sided Chebyshev's inequality

$$P(\mathbf{X} \leq \mu - z\sigma) \leq \frac{1}{1+z^2},$$

where μ and σ are, respectively, the mean and the standard deviation of $\mathbf{X}$ for this decision maker. Notice that for $z = 2$ this gives an upper bound for $P(\mathbf{X} \leq \mu - 2\sigma)$ of 20%, an estimate made without making any assumptions about the shape of the distribution of $\mathbf{X}$.

Remark 4 One can say that, based on the above, there is a precise sense in which, for the purpose of coming up with upper bound estimates of the lower tail of the distribution of beliefs of a CARA decision maker, *the certainty equivalent is to the mean of the belief distribution as the risk tolerance coefficient is to the standard deviation of the belief distribution*. This could be of importance when developing real-world applications that involve elicitation of subjective beliefs from one or more decision makers, as in Myerson (2005).

3 Applications

3.1 Value at risk

Value at risk is a measure used to estimate how the value of an asset or of a portfolio of assets could decrease over a certain time period under usual conditions. Usual conditions in this context is defined to mean "under most circumstances," which in turn is described with respect to a probabilistic confidence level, usually 95 or 99%. The challenge in many applied contexts is to come up with estimates of the value at risk for a decision maker even if we do not know the shape of the decision maker's probabilistic beliefs.

Formally, the *value at risk at a confidence level of* p of a risky prospect $\mathbf{X}$ worth CE to a decision maker is the maximum loss L that the decision maker expects to incur with probability p. It can be calculated by the following formula:

$$CE - (1-p)\,th\ percentile\ of\ \mathbf{X}.$$

It turns out that the following is true:

Theorem 3 *Suppose* $\mathbf{X}$ *is a risky prospect valued at* CE *by a decision maker with invertible utility function* u *with* $\overline{u} = 0$. *Then the value at risk,* L, *at a confidence level of* p *for this decision maker must satisfy*

$$L \leq CE - u^{-1}\left[\frac{u(CE)}{(1-p)}\right]. \tag{2}$$

Proof From Theorem 1 and from the fact that $u(CE-L) < 0$ it follows that

$$u(CE-L) \geq \frac{u(CE)}{P(\mathbf{X} \leq CE-L)}.$$

Since u is increasing and invertible, and setting $P(\mathbf{X} \leq CE-L) = 1-p$, then

$$CE-L \geq u^{-1}\left[\frac{u(CE)}{1-p}\right],$$

which gives a lower bound on the $(1-p)$th percentile of the distribution of $\mathbf{X}$ according to the decision maker. Rearranging gives the desired result. □

Notice that for the case of decision makers with a constant absolute risk aversion coefficient given by $1/r$ expression (2) reduces to

$$L \leq -r\ln(1-p)$$

Example 1 Suppose $\mathbf{X}$ is a risky prospect valued at 100 by a decision maker with constant risk tolerance parameter given by $r = 10$. Then an analyst that knows this about the decision maker will know that the value at risk at a confidence level of 95% for the decision maker will be below 30. That is, the analyst will know that the decision maker considers his losses to be below a certain threshold with probability 95%; this threshold the analyst knows will be below 30.

Remark 5 It is standard in many applications to use the variance σ^2 of a risky propect $\mathbf{X}$ as a way to measure its riskiness. One potential drawback to this method is that it is not always easy for an analyst to adequately estimate the variance of $\mathbf{X}$ that is implicit in the probabilistic beliefs of the decision maker.[3] The methods used above to estimate bounds to the value at risk of a risky prospect can also be used to develop upper and lower bounds for the variance and the half-variance of a risky prospect, bounds that depend on the decision maker's attitudes towards risk, on the certainty equivalent of the risky prospect, and on its risk premium.

3.2 Option valuation

Having upper estimates of the probabilities of tail events for a decision maker can be of use to see how this decision maker would value financial instruments whose payoffs are tied to the occurrence of those events. In short, they can be of use for the valuation of options and other financial derivatives.

The ability to use the inequalities developed in the present paper to put *expected utility bounds* on option values is potentially very valuable, as it is known that the parametric estimation of those tail probabilities is a challenging task. Another potential

[3] See, e.g., the discussion in Morgan and Henrion (1990). See also Myerson (2005).

problem is that sometimes we are dealing with one-of-a-kind projects for which there is not even any previous data to use in an estimation process.

The use of upper bounds to the probability of those tail events sidesteps completely the tail estimation problem, as it avoids making distributional assumptions. This, of course, can come at a cost if one ends up with bounds to the option values that are not very informative. Whether this is so or not is an empirical question that deserves attention in its own right.

What follows illustrates the kinds of bounds on the value of options that arise from a judicious application of Theorem 1 to the valuation of out-of-the-money put options.[4]

The setup is, again, a risky prospect $\mathbf{X}$ that takes values in a set of possible outcomes $\mathcal{C}$. The decision maker evaluates $\mathbf{X}$ with an increasing utility function u and with respect to beliefs given by a distribution P, unknown to the analyst. All payoffs and prices in this Section are expressed in present value terms.

I am interested in how much this decision maker would value a contract that gives the decision maker the right but not the obligation to sell risky prospect $\mathbf{X}$ at a predetermined (strike) price S. I am thus interested in the value for the decision maker of the risky prospect

$$\mathbf{T}_S = \max\{S - \mathbf{X}, 0\},$$

commonly known as a put option on $\mathbf{X}$ with strike price S. To find the value of $\mathbf{T}_S$ for the decision maker one has to find the certainty equivalent of $\mathbf{T}_S$, that is, the price Q_S such that

$$u(Q_S) = Eu(\mathbf{T}_S).$$

The following result will be needed in what follows.

Lemma 4 *Suppose* $\mathbf{X}$ *is a risky prospect to be valued by a decision maker with utility function* u. *Let* $\mathbf{T}_S$ *be a put option on* $\mathbf{X}$ *with strike price* S. *Then*

$$Eu(\mathbf{T}_S) \leq u(S)\,P(\mathbf{X} \leq S) + u(0)\left[1 - P(\mathbf{X} \leq S)\right] \tag{3}$$

Proof

$$u(\mathbf{T}_S) = u(\max\{S - \mathbf{X}, 0\}) = u(S - \mathbf{X})\,1_{\{\mathbf{X} \leq S\}} + u(0)\,1_{\{\mathbf{X} \geq S\}}$$

Since u is increasing, $u(S - x) \leq u(S)$, so

$$u(\mathbf{T}_S) \leq u(S)\,1_{\{\mathbf{X} \leq S\}} + u(0)\,1_{\{\mathbf{X} \geq S\}},$$

and taking expectations yields the desired result. □

[4] A similar exercise done on the valuation of call options produces bounds on the option values that are not very informative, and it is therefore of little interest.

Even if an analyst knows the utility function of the decision maker Eq. (3) is of little help if one has no information about the beliefs held by the decision maker. This is where an estimate of $P(\mathbf{X} \leq S)$ can be of help. Such estimate is available for $S < CE$, that is, for options that are *out of the money*.

In what follows I assume that u is bounded from above and normalize the origin of u in such a way that $u(x) \leq 0$ for all $x \in \mathcal{C}$ and the units of the the utility function in such a way that $u(0) = -1$.[5] Also, let $\underline{P(\mathbf{X} > S)} = 1 - \frac{u(CE)}{u(S)}$. From Theorem 1 it follows that one can interpret $\underline{P(\mathbf{X} > S)}$ to be a lower bound on the probability that the decision maker assigns to $\mathbf{X}$ taking values greater than S.

Theorem 5 *Suppose* $\mathbf{X}$ *is a risky prospect valued at* CE *by a decision maker with utility function* u *with* $\overline{u} = 0$ *and for which* $u(0) = -1$. *Then, for* $S < CE$,

$$Eu(\mathbf{T}_S) \leq Eu(\mathbf{X}) - \underline{P(\mathbf{X} > S)}. \tag{4}$$

Proof From Lemma 4 we have that

$$Eu(\mathbf{T}_S) \leq u(S)\,P(\mathbf{X} \leq S) + u(0)\left[1 - P(\mathbf{X} \leq S)\right],$$

while Theorem 1 gives us a bound on $P(\mathbf{X} \leq S)$:

$$P(\mathbf{X} \leq S) \leq \frac{u(CE)}{u(S)}.$$

Combining these two expressions and rearranging we get

$$Eu(\mathbf{T}_S) \leq \frac{u(CE)}{u(S)}u(S) - \left(1 - \frac{u(CE)}{u(S)}\right),$$

that is,

$$Eu(\mathbf{T}_S) \leq Eu(\mathbf{X}) - \underline{P(\mathbf{X} > S)}.$$

□

Expression (4) then reads as follows: the expected utility of holding the put option cannot exceed the expected utility of holding the original risky prospect, minus the analyst's lower estimate on the probability that the option will expire worthless.[6] This expression can be used to find an upper bound to the value of the option, since $u(Q_S) = Eu(\mathbf{T}_S)$ and $u(CE) = Eu(\mathbf{X})$. I record the formula for obtaining such bound below.

[5] Notice that I am not assuming in this case that -1 is the lower bound of u on $\mathcal{C}$.

[6] Of course, discussing expression (4) in this way makes sense specifically for the normalization of the utility function chosen above.

Table 1 Upper bound on the price of put options given a strike price of S and constant risk tolerance r for a risky prospect worth 100 to the decision maker

r	S						
	60	65	70	75	80	85	90
8	0.05	0.1	0.19	0.36	0.69	1.33	2.70
10	0.18	0.31	0.51	0.86	1.45	2.52	4.59
15	1.06	1.51	2.16	3.12	4.56	6.85	10.77

Corollary 6 *Suppose* **X** *is a risky prospect valued at* CE *by a decision maker with invertible utility function* u *with* $\overline{u} = 0$ *and for which* $u(0) = -1$. *Then, for* $S < CE$,

$$Q_S \leq u^{-1}\left[u(CE) - \left(1 - \frac{u(CE)}{u(S)}\right)\right]. \tag{5}$$

Notice that for the case of decision makers with constant risk tolerance r expression (5) reduces to

$$Q_S \leq -r \ln\left[1 + e^{-\frac{CE}{r}}\left(1 - e^{\frac{S}{r}}\right)\right]$$

Example 2 Suppose **X** is a risky prospect valued at 100 by a decision maker with constant risk tolerance parameter. Table 1 shows upper bounds on the values of put options of different strike prices for varying degrees of tolerance for risk.

For example, a decision maker with constant risk tolerance coefficient given by $r = 10$ that values **X** at 100 will value a put option on **X** with strike price 65 at some level that the analyst knows will never be above 0.31. Another way to put this is as follows: given that the decision maker values **X** at 100 there is no probabilistic belief that the decision maker could have about **X**, no matter how pessimistic it may be, that would justify the analyst believing that the decision maker would pay more than 0.31 for this option.

3.3 Credit risk

The expected utility bounds developed in this paper can be used for estimating the probability that an entity with limited liability will default on its debt. They can also be used for estimating the value of this debt to some decision maker.

The setup is one in which there is an entity with limited liability that owns the proceeds of risky prospect **X**. The entity has debts by an amount equal to $B > 0$ that are due at the moment the value of **X** is realized. As before, a decision maker evaluates **X** with an increasing utility function u and with respect to beliefs given by a distribution P. The entity and the decision maker in this setup may be the same, although this is not necessary. All payoffs and prices in this section are expressed in present value terms.

The first result of this section is an upper bound on the probability that the entity will default on its debt.

Theorem 7 *Suppose* **X** *is a risky prospect valued at* CE *by a decision maker with utility function* u *with* $\overline{u} = 0$. *Suppose the entity that owns* **X** *has* $B > 0$ *in debts, with* $B < CE$. *Then the probability,* P^d, *for this decision maker, that the entity will default on its debt satisfies*

$$P^d \leq \frac{u\,(CE)}{u\,(B)}$$

Proof The entity enters default when the realized value of **X** is not sufficient to cover the decision maker's obligations, that is, when $\mathbf{X} < B$. Hence, $P^d = P\,(\mathbf{X} \leq B)$ and an application of Theorem 1 for the case where $k = CE - B$ yields the desired result. □

Example 3 Suppose **X** is a risky prospect valued at 100 by a decision maker that has a constant risk tolerance parameter given by $r = 20$. If the entity that owns **X** owes 40 to creditors then this decision maker believes that the entity will default on its debt with probability P^d, a probability that the analyst knows is below 4.98%.

Due to limited liability, owning a risky prospect **X** with B in debts is just like owning a call option on **X** with strike price B. This fact, together with the put-call parity, implies that the value of the risky debt equals the value of riskless debt of identical face value minus the value of a put option on **X** with strike price B, that is,

$$V_B = B - Q_B, \tag{6}$$

where Q_B is the certainty equivalent for this decision maker of $\mathbf{T}_B = \max\{B - \mathbf{X}, 0\}$. One can therefore find lower bounds on the value of the risky debt to a particular decision maker using the results from Sect. 3.2.

Theorem 8 *Suppose* **X** *is a risky prospect valued at* CE *by a decision maker with invertible utility function* u *with* $\overline{u} = 0$ *and for which* $u\,(0) = -1$. *If the entity that owns* **X** *has debts given by* $B < CE$ *then the decision maker values this debt by an amount* V_B *that satisfies*

$$V_B \geq B - u^{-1}\left[u\,(CE) - \left(1 - \frac{u\,(CE)}{u\,(B)}\right)\right].$$

Proof Combine Eq. (5), with strike price equal to B, and Eq. (6) to get the desired result. □

Example 4 Suppose **X** is a risky prospect valued at 100 by a decision maker that has a constant risk tolerance parameter given by $r = 30$. If the entity that owns **X** also owes 40 to creditors then this decision maker values this debt at a level V_B, a level that the analyst knows is above 36.9.

4 Discussion

To put the expected utility inequalities presented in this paper in proper perspective it is important to reflect on the strengths and weaknesses of the approach undertaken here regarding the elicitation of subjective beliefs.

The main weakness that the method proposed above has is that it requires the analyst to know the decision maker's attitudes towards risk, that is, the decision's maker (Bernoulli) utility function. This is, of course, problematic, as this information can be as difficult to elicit as the subjective beliefs themselves. I wish to argue that whether this is a serious problem or not depends on the intended application of the results presented here.

To make this point consider that one can argue, from a strict decision theoretic point of view, that the same procedure that is used to elicit the decision maker's utility function could be used to assess both the utility function and the subjective beliefs. For example, it is known that to recover the decision maker's utility function $u(x)$ defined, say, over the interval $\left[\underline{x}, \overline{x}\right]$ with normalization $u\left(\underline{x}\right) = 0$ and $u(\overline{x}) = 1$ all that is required is that we find the "probability equivalent of x," namely, the value $u(x)$ that makes the decision maker indifferent between x and the binary lottery $\left[\overline{x}, u(x); \underline{x}, (1 - u(x))\right]$. An identical approach could be used to elicit the subjective cumulative distribution function $F(x)$ of $\mathbf{X}$, namely, by finding the value $F(x)$ that makes the decision maker indifferent between

$$\left[\$100, F(x); \$0, (1 - F(x))\right]$$

and

$$\left[\$100 \text{ if } \mathbf{X} \geq x;\ \ \$0 \text{ if } \mathbf{X} < x\right],$$

and by allowing x to vary between $\underline{x}$ and $\overline{x}$. If one did this of course it would not be necessary to estimate bounds on F by using the expected utility inequalities developed in this paper because one could recover the entire distribution F to begin with in the same way the utility function is recovered.

On the other hand, the main advantage of the method proposed in this paper is that, once the analyst has elicited the utility function in the context of a particular risky prospect $\mathbf{X}$, this same information can be reused again and again in the investigation of the subjective beliefs that the decision maker would have about many other risky prospects, and not just about the one that was employed in the elicitation of the utility function.

In light of the above, it is appropriate to consider the results here to be most useful when we have already recovered information about the decision maker's attitudes towards risk, and we wish to know how optimistic or pessimistic this decision maker feels about the outcomes in a wide variety of different decision problems. This approach does makes sense for many applications where, for example, the attitudes towards risk have either been calibrated or estimated econometrically from existing data (e.g., older, recently expired contracts) and one is interested in the beliefs that the decision maker has over a series of different risky prospects (e.g., newer, unexpired contracts) for which we have information about valuations (i.e., the certainty equivalents) that perhaps come from the comparison of different observed prices.

5 Conclusions

In this paper I develop *expected utility bounds* to the tails of the probabilistic beliefs of a risk averse decision maker over a risky prospect based on economic magnitudes such as the certainty equivalent and on the decision maker's attitudes towards risk. I also develop applications of these expected utility bounds to several economic problems, such as estimation of risk measures from observed data, option valuation and the study of credit risk. The bounds are very general in that they require no knowledge about the functional form of the probabilistic beliefs held by the decision maker.

These bounds can be used in one of two ways: (i) to generate estimates of certain unobservable variables, based on what is observable, and (ii) as an intermediate step in a more elaborate theoretical argument. Consequently, they should be of interest to both theoretical and empirical researchers alike.

References

LiCalzi, M.: Upper and lower bounds for expected utility. Econ Theory **16**, 489–502 (2000)

Morgan, M., Henrion, M.: Uncertainty: a guide to dealing with uncertainty in quantitative risk and policy analysis. Cambridge: Cambridge University Press (1990)

Myerson, R.: Probability models for economic decisions. Belmont: Thomson (2005)

Rothschild, M., Stiglitz, J.: Increasing risk. I. A definition. J Econ Theory **2**, 225–243 (1970)

Rothschild, M., Stiglitz, J.: Increasing risk. II. Its economic consequences. J Econ Theory **3**, 66–84 (1971)